From Cicero to Peano

Milestones on the Road of Scientific Latin



Julian Schennach

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Latin Texts with
Running Vocabulary and Commentary

Edited, with introduction and notes, by
Julian Schennach

PIXELIA PUBLISHING

From Cicero to Peano: Milestones on the Road of Scientific Latin

First Edition

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Paperback ISBN: 978-1-967785-13-1
Ebook ISBN: 978-1-967785-14-8
Published 2026 by Pixelia Publishing (Williamsburg, VA, USA)
`https://pixeliapublishing.org`
Cover design: Julian Schennach
Front image: Photo of the reverse of a Fields Medal made by Stefan Zachow for the International Mathematical Union (IMU). Public domain.
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Interior Font: Bembo-like font, based on Cardo, provided by the Latex package fbb.

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In memory of my grandfather Martin S. Schennach

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Acknowledgments

The creation of this reader would not have been possible without the enthusiastic support of the entire Pixelia Publishing editorial board: Thomas Hendrickson, Ben Wiebracht, Anna Pisarello, John Lanier, and Caedmon Haas. I want to express particular gratitude towards Dr. Hendrickson, for thorough and constructive feedback on all parts of the text, as well as more generally his indefatigable encouragement and advice throughout my Latin journey at Stanford OHS; Dr. Wiebracht, for his thoughtful suggestions in revising the book and author introductions; and Magister Lanier, for guidance in the formatting and typesetting of the reader. I would also like to thank Josh Carlson, Christine Gosnay, and Tomohiro Hoshi for the financial and institutional support of Pixelia Publishing.

This work has been a significant undertaking and a big part of my life for nearly two years. I must thank Dr. Lucio Mare's "History and Philosophy of Science" course for helping to spark my interest in the evolution of scientific thought and worldviews over the millennia. Similarly, Nicholas Ostler's *Ad Infinitum* instilled in me an appreciation for the historical and crosslinguistic background of Latin and inspired my study of Ancient Greek.

The composition of this reader has been guided by two existing anthologies that also aim to familiarize students with post-Classical Latin and some of its diverse genres: Mark Riley's *The Neo-Latin Reader: Selections from Petrarch to Rimbaud*; and, turning to the history of science itself, Marcelo Epstein and Ruth Spivak's *The Latin of Science*. I expect that this reader and its predecessors, differing as they do in their selections and their target audiences, will complement each other. Another notable anthology of important Neo-Latin works is Milena Minkova's *Florilegium Recentioris Latinitatis*, written entirely in Latin.

More generally, I want to credit Dickinson College Commentaries, Faenum Publishing, and Geoffrey Steadman (geoffreysteadman.com) for helping to popularize the format of open-access editions of Greek and Latin texts with on-page vocabulary and grammatical commentary.

Of course, many thanks are due to my friends and family for their unwavering support over the course of the past few years, and to my Ancient Greek and Latin reading buddies Sammy and Gene.

Introduction

1 The Purpose of this Reader

Latin is traditionally taught through the study of a few Classical Roman authors such as Cicero or Vergil. Yet, Latin has had a significant influence extending far beyond the duration of the Roman Empire, in large part due to its role as the main language of science from the late Middle Ages to the 18th century. The number of Latin publications in this field and period dwarfs the number of Classical texts that have survived into our time. This reader aims to familiarize students with Latin scientific literature, thus providing Latin instructors with an array of interesting material to complement more traditional Classical texts and to help students better appreciate the significance of Latin throughout history. After all, as readers will discover, the sheer impact of scientific Latin works on modern scientific thought and today's technological world is unquestionable.

High school and college curricula can often impose unnatural divides between the humanities and the sciences. This reader's broadening of the typical Latin corpus of pedagogical texts offers an invitation to a whole new set of students, particularly those with an affinity for STEM who might otherwise only take the bare minimum of language classes required to graduate. This reader's texts are thus selected with the intent to bring together an academically diverse class of students, with an eye towards instructors whose expertise may align more closely with the language than the scientific content.

Even in areas of science that have since been extensively studied, it is nevertheless important to revisit, if only on occasion, their histori-

cal foundations. For instance, the appropriate way to translate Newton's Latin formulation of his first law of motion has periodically reignited debate, with news accounts of the discussion recently even reaching *Scientific American* (Pappas, 2023). The history of science in general thus demands interdisciplinary skills. They range from an ability to read Latin or other relevant scientific languages, to knowledge of scientific and mathematical theories (to understand what the Latin is communicating), to a familiarity with the society and academic practices of the past. This reader hopes to give a taste of the depth and breadth of this subject.

Since "science" itself is a time-dependent concept, this collection includes scientific texts that modern readers might not immediately recognize as such. Early scientists were more akin to philosophers and, during most of history, scientists tended to be mathematicians as well. This reader is centered on early modern science and mathematics (Copernicus, Kepler, Leibniz, Merian, Newton) complemented with some Classical antecedents (Cicero, Lucretius), and important successors (Euler, Gauss, Peano). These texts seek to illustrate the transition from Classical Latin (e.g. Cicero, Lucretius) to the subsequent scientific Latin while covering the whole time-span between the rise of scientific Latin (e.g. Copernicus) and its decline (e.g. Peano).

In order to bridge the gap between STEM-oriented students and language-oriented teachers, this collection focuses on texts that are historically significant and yet not too technical. No advanced knowledge of science or mathematics is required, although optional info boxes for about half of the authors in this reader do provide additional historical or technical information for interested readers.

(See the next two pages for a sample infobox tracing an interesting interplay between scientific knowledge and Latin writing, set at the height of the Scientific Revolution.) The chosen texts are sometimes slightly off the beaten path, in some ways unusual, or even humorous — a reminder that science has always been motivated by the spirit of adventure and of play. There are also some recurring themes: the search for a universal language, changes in worldview, first-of-a-kind manuscripts, etc. To aid in the compilation of a preliminary syllabus, we provide brief summaries of our excerpts and explanations for their inclusion below. The author introductions preceding each set of excerpts elaborate on them and situate the texts in the wider context of the author's life and the history of science. Someone interested in the sciences and mathematics is likely to be familiar with most of the names in this reader, but probably not with our slightly unorthodox and counter-canonical choice of texts.

Marcus Tullius Cicero (106 BCE – 43 BCE): In the first excerpt, from the *Tusculānae Disputātiōnēs*, Cicero celebrates his rediscovery of the tomb of Archimedes, an engaging story that provides not only a direct connection back to the achievements of Ancient Greek mathematicians, which may otherwise be obscured by our focus on works composed in Latin, but also a window into the Romans' opinions of their mathematically talented neighbors. The second excerpt, from the *Dē Nātūrā Deōrum*, is a humorous and hyperbolic refutation of the premise that the world, or God, is better than any other object in every dimension, which foreshadows certain objections to later ontological arguments by Anselm of Canterbury and others.

Sample Infobox: Galileo's Anagrams & Kepler's "Solutions"

From 1610 to 1613, Galileo Galilei observed with the aid of his telescope what he would interpret as two important pieces of evidence in support of the Copernican, heliocentric model over competing geocentric ones: first, that Venus had phases like the Moon, with its illuminated side periodically facing the Earth; and second, that Saturn exhibited an almost oblate and ever-changing form (which Christiaan Huygens would later explain as an appearance produced by the varying orientation of Saturn's rings relative to the Earth). In the geocentric Ptolemaic system, which was widely accepted at the time, Venus always remains situated between the Sun and Earth, preventing it from ever becoming full from the perspective of the Earth. Furthermore, the Aristotelian worldview that serves as the philosophical foundation for the Ptolemaic model assumes the perfection of all superlunary celestial bodies, a perfection that manifests in their regular orbits and their sphericity, which Saturn now seemed to violate.

Having just released his *Sīdereus Nūncius* in 1610, Galileo was unable to immediately incorporate these discoveries in an upcoming publication, but he wanted to avoid contentious priority disputes in the future, of which there is no shortage in the history of science and mathematics. Galileo's solution was to encode the announcements of his observations in anagrams sent out to fellow astronomers, who, once he formally published his claims and revealed the decryptions, would then have to recognize him as the first to make these discoveries, even if some other astronomer managed to publish them first. As his first anagram, he scrambled the phrase *altissimvm planetam tergeminvm observavi* ("I observed the highest body" — i.e. the last planet Saturn — "triform" — i.e. with a three-body system that he later sketched as oOo) into *smaismrmilmepoetalevmibvnenvgttavrias*. Galileo's second anagram was even more ingenious, with the plain text *Cynthiae figvras aemvlatvr mater amorvm* ("the mother of love emulates the shapes of Cynthia,"

i.e., the personification of the Moon). Here, the encryption *haec immatvra a me iam frvstra legvntvr o y* ("these immature things are now read by me in vain o y") actually forms a comprehensible Latin sentence, and one that cheekily mocks the efforts of his reader to decode it. In August and December of 1610, Galileo sent each of the two anagrams to Johannes Kepler, who used some of his theological beliefs and astronomical studies to engineer incorrect, yet surprisingly prescient decipherings (Marcus and Findlen, 2019): Convinced that the number of moons possessed by the outer planets of the solar system should follow a geometric series, Kepler suspected that Galileo determined that Mars had two moons, in accordance with the Earth's single moon and Jupiter's four known Galilean moons (Goehring, 1981). Therefore, he rearranged the given anagram into the dactylic hexameter line *salve vmbistinevm geminatvm Martia proles* (which can be translated as "greetings, twinned bystander, progeny of Mars" if *umbistineum* is taken as a Germanism) (Marcus and Findlen, 2019).

In his studies on the *anima mōtrīx*, his early version of the Newtonian gravitational force, Kepler theorized that a celestial body must rotate on an axis to generate the circular guiding lines of force that would sustain the orbit of a satellite (Goehring, 1981). Once again based on the existence of Jupiter's moons, he inferred that Galileo must have found evidence for its revolution, perhaps in the form of moving markings on the planet — hence the solution *nam Iovem gyrari macvla hem rvfa testatvr* ("hah, indeed it is attested through a red spot that Jupiter rotates"), which Kepler admitted himself to be clunky (Marcus and Findlen, 2019). Thus, Kepler managed to correctly predict, albeit with incorrect reasoning, the discovery of Jupiter's Great Red Spot and of the moons of Mars, Deimos and Phobos.

Many thanks to the Archimedes Laboratory Project webpage for inspiring the research into these anagrams:

`https://www.archimedes-lab.org/latin.html`

Titus Lucretius Carus (possibly 99 BCE – 55 BCE): This collection's excerpt of Lucretius's *Dē Rērum Nātūrā* captures the poet's mastery of dactylic hexameters and his clear and persuasive *ad absurdum* argumentative structure. Students will also relate to his description of the difficulties of translation (in his case, of Greek philosophical terms into Latin). In the spirit of the Ancient Greek philosophy of Epicureanism that he followed, Lucretius also expresses a driving principle behind all science, by warning against the dismissal of supposedly unexplainable natural phenomena as the arbitrary and terrifying acts of gods.

Nicolaus Copernicus (1473 CE – 1543 CE): The unassuming title of the *Commentāriolus*, "little commentary," underplays the historical significance of the work, as it contains Copernicus's first description of his heliocentric model, which would shatter the prevailing Aristotelian worldview and eventually usher in a new, much more mathematical one. This text precedes his magnum opus *Dē Revolūtiōnibus Orbium Coelestium* by over three decades. In addition to laying out the axiomatic foundations of heliocentrism, the introduction excerpted in this reader references and critiques the ancient geocentric models of Eudoxus and Ptolemy, making it a fitting transition from the classical works above.

Johannes Kepler (1571 CE – 1630 CE): Kepler is well known for his scientific accomplishments, but he also holds an important place in the history of literature. His *Somnium*, excerpted in this volume, is often recognized as the very first work of science fiction. This dream about a voyage to the Moon highlights the satellite's geography and astronomy from the per-

spective of its fictional inhabitants. Despite the fantastical elements of witches and demons, the work is rigorously engaged with the science of its day, as the many footnotes scattered throughout attest. Our excerpt explains how humans could endure a trip to the Moon; interspersed throughout are some self-deprecating jokes regarding who should and should not undertake such a demanding voyage, with philosophers being too sedentary and Germans too gluttonous.

Gottfried Wilhelm Leibniz (1646 CE – 1716 CE): The *Dē Arte Combīnātōriā*, Leibniz's earliest published work, discusses combinatorics[1] and, in the first of the two passages provided in this reader, his ideas for a universal language or *charactēristica ūniversālis*, which he would repeatedly revise throughout his whole life. After rejecting the purely numerical approach adopted by other mathematicians of his time, Leibniz suggests that combinatorics can provide a practical — and computational — implementation of a universal language, where even just a few symbols of basic meaning (akin to Egyptian or Chinese pictographs) can be combined to express any conceivable idea. The second half of our excerpt contains a definition of factorials and a fun application of them to a classical problem: How many times does a host need to distribute n guests across n seats in order to have tried all possible seating arrangements?

Maria Sibylla Merian (1647 CE – 1717 CE): As a naturalist and entomologist, Merian studied the births and metamorphoses of insects at a time when the theory of spontaneous generation remained quite widespread. Remarkably for a divorced woman, Merian traveled alone with her daughter to

[1] Combinatorics is an area of mathematics centered on counting the arrangements of objects given certain restrictions.

Suriname to study the region's plant and insect life, collecting the information she gathered into the book *Metamorphōsis Īnsectōrum Surinamēnsium*, which was published in both Dutch and Latin. The first passage touches on the colonial backdrop of modern science through its description of the *flōs pāvōnis* or peacock flower, which Merian states was often used by indigenous slaves to abort a pregnancy and spare their potential children from slavery. The second excerpt from the text treats the interactions between the citrus plant, hummingbirds, and different species of spiders and ants in a holistic way that contrasts with the mechanistic philosophy of her contemporaries. In doing so, Merian's work foreshadows the views of 19th-century biologists such as Alexander von Humboldt and the modern concept of an "ecosystem." The beauty of Merian's carefully composed and scientifically accurate illustrations cannot be overstated, and several of them are reproduced here.

Isaac Newton (1643 CE – 1727 CE): Newton's *Philosophiae Nātūrālis Prīncipia Mathēmatica* may be the most famous scientific work ever published in Latin. However, the familiar and enlightening cannonball thought experiment included here, which depicts the consistent nature of universal gravitation on local and celestial scales, actually originates from a draft version of the *Prīncipia Mathēmatica* that Newton himself never intended to release. This excerpt, accompanied by the memorable diagram, serves as the conclusion to the astronomical subtheme of this reader.

Leonhard Euler (1707 CE – 1783 CE): Among Euler's many advanced mathematical accomplishments, from number theory to calculus, from music theory to engineering, his solu-

tion to the Königsberg bridge problem is a gem of simplicity that merits a place in this reader for initiating the field now known as graph theory. Euler's proof that it is impossible to cross all of the seven bridges surrounding two central islands of Königsberg (of which only two have survived) exactly once requires no prior knowledge of special notation, and the article nicely generalizes the result to any arrangement of islands and bridges, with the full proofs summarized in the commentary for the sake of concision.

Carl Friedrich Gauss (1777 CE – 1855 CE): The *Disquīsītiōnēs Arithmēticae*, authored by Gauss at the age of twenty-four, is considered the first comprehensive work on number theory and modular arithmetic (which studies the properties of remainders of divisions) and is responsible for much of the notation presently employed in these areas of mathematics. The excerpts from Gauss's preface to the *Disquīsītiōnēs* highlight Ancient Greek and contemporary 17th- and 18th-century contributions to what would now be classified as number theory, with Gauss acknowledging the fact that he later found out that many results in his book were actually independent rediscoveries. For some accessible concrete mathematical results, we include the first few paragraphs of the main body of the *Disquīsītiōnēs*, where Gauss defines the notion of congruences and proves the book's first theorem.

Giuseppe Peano (1858 CE – 1932 CE): While our reader opens with Classical Latin at its peak as embodied in the language of Cicero, this final selection captures scenes of the twilight of Neo-Latin from Peano's *Dē Latīnō Sine Flexiōne*. Here, the mathematician devises a simplified version of Latin that

preserves its vocabulary but strips it of all inflections (conjugations and declensions) so as to be immediately comprehensible to any speaker of a Romance language. Guided by Leibniz's work on a *charactēristica ūniversālis*, Peano intended his artificial language to be perfectly adapted to the needs of scientific, mathematical, and philosophical discussions. A brief excerpt from the companion article *Prīncipiō dē Permanentia* demonstrates the utility of the language in a debate on best practices in mathematical notation, another major interest of Peano.

2 What is Science?

Science and mathematics, which form the basis for the selections in this reader, enjoy a close symbiotic relationship, most clearly in their modern forms: Mathematicians may imagine objects that are initially interesting only from a theoretical perspective and yet eventually become invaluable parts of the repertoire of scientists. The physicist Eugene Wigner has referred to this pipeline as the "unreasonable effectiveness of mathematics." Imaginary numbers are a particularly striking example of this phenomenon. They were first conceived of in the 16$^{\text{th}}$ century and were disparagingly named by René Descartes after their supposed impossibility in the real world, but have since proven themselves essential in understanding many of the rather unintuitive aspects of 20$^{\text{th}}$-century quantum mechanics. Conversely, the needs of scientists often encourage novel developments in mathematics, as demonstrated by the invention of calculus by the physicists Newton and Leibniz or the contributions to matrix theory by the quantum physicist Werner Heisenberg.

From a philosophical perspective, mathematics seems to quite straightforwardly designate any field involving the definition of objects or derivation of truths from more elementary concepts. This gradual constructive approach is arguably reflected in the etymology of the term "mathematics," which is derived from the Greek word for "learning." Admittedly, mathematics has entertained its fair share of philosophical debates, mostly on its logical foundations and on the reality of mathematical concepts. On the other hand, science (from the Latin *scientia*, with the broad sense of "knowledge"[2]) has posed some of its greatest difficulties for philosophers at the basic level of its definition. At first glance, it would seem reasonable to define an area of research as "scientific" if it aims to provide explanations for the workings of the universe, while also allowing for the possibility that those explanations will be proven false by new data, often procured through experiments or fresh observations. This ultimate deference to the data is one of the principles behind the notion of falsifiability proposed by the 20th-century philosopher Karl Popper (Popper, 2005). However, this perspective is not applicable to theoretical parts of science that are more or less detached from work in the laboratory, and it may create a presentist bias, unfairly dismissing ancient or medieval non-experimental approaches to science that were at the time deemed rigorous and testable and may be of interest to historians.

The 20th-century philosopher of science Imre Lakatos pointed out another problem with this understanding of science, namely the fact that scientists can be reluctant to throw away long-standing theories just because of a few new experimental findings. In response,

[2]"Scientist" as a word used to describe someone studying science is a relatively recent English coinage, by William Whewell in 1833 upon the request of none other than the Romantic poet S. T. Coleridge (Ross, 1962).

he suggested instead that what qualifies as science evolves over time. A theory's status depends on its current ability to not only address known natural phenomena, but also, more importantly, generate new knowledge with the help of the techniques it offers as a unified toolkit or "programme." A theory or programme as a whole is "degenerating" — i.e. no longer "progressive" or productive — if it must be continuously retrofitted to account for disagreeing results or new theoretical arguments (Lakatos, 1978). Note that some components of a theory, such as its methodology, may nevertheless remain progressive in isolation and be reused. Ultimately, Lakatos's framework is more accepting of ancient scientific models and is aligned with our selection of scientific texts in this reader.

Let us now briefly outline the development of science and mathematics in the areas sampled in the excerpts of this reader, with an acknowledgment that our focus on Latin texts leads to an emphasis on the Western tradition and the Classics-adjacent world. Through communication and travels across the Mediterranean in both directions, the Ancient Babylonians and Egyptians, themselves known for their extensive astronomical and architectural applications of algebraic and trigonometric algorithms, helped influence (directly or indirectly) early Ancient Greek mathematicians in the 6th century BCE. Thales of Miletus in particular is often recognized as a key initiator of the transition away from mythological explanations of natural phenomena, found in, for instance, the Homeric epics, to continuously revised and debated theories on the underpinnings of nature (Lloyd, 1970). This Ancient Greek natural philosophy progressed alongside and in partnership with a methodological framework for philosophy in general, namely logic. In the 4th century BCE, Aristotle formalized the study of logic in his *Organon*, partly

with insight derived from the proof techniques by mathematicians, and applied it to his epistemology and the extensive and internally consistent natural philosophy (physics, cosmology, teleology, biology, etc.) comprising what is now known as the Aristotelian worldview. While not an experimentalist in the modern sense, Aristotle did encourage an active observational approach previously employed by historians and physicians, one that has been retained as an important tool in biological research, where it may not always be ethical or even possible to study phenomena in controlled, experimental settings (Lloyd, 1970).

Mathematics also acquired a close connection to philosophy in the Ancient Greek world beyond its significance for logic, as exemplified by the Pythagoreans in the 6th century BCE. They were a sect divided into the ἀκουσματικοί, who "listened" and adhered to the spiritual teachings of their founder Pythagoras, and the μαθηματικοί, who "learned" from, and then were able to engage in, his mathematical teachings. These two areas of focus intersected through the belief that the universe was generated from the fundamental unit of "one" and that musical ratios and the harmony of the planets echoed the divinity of the integers (Kahn, 2001). A slightly less mystical expression of the ideality of mathematics in metaphysics is due to Plato, who identified numbers and geometric shapes as being among the perfect forms that all real-life objects attempt to imitate. This perception of mathematics had a particularly lasting influence on astronomy, where planetary models were supposed to not only predict the positions of the celestial bodies, but also satisfy certain criteria for elegance, e.g. the presence of circular motions, that would complement the regularity of the heavens. The problem of finding a proper balance between accuracy and

elegance was called "saving the phenomena," or, in Greek, σῴζειν τὰ φαινόμενα, which would eventually prompt Copernicus to propose his heliocentric model (Lloyd, 1970). In contrast to its influence on astronomy, mathematics was generally considered unsuitable for the realm of what were, at the time, considered imperfect terrestrial phenomena, although important exceptions can be found in, for instance, Aristotle's rough quantitative descriptions of projectile motion, Archimedes's mechanics (levers and buoyancy), and Euclid's geometric treatment of light. On the Roman side, Vitruvius emphasized the importance of mathematical harmonies for architectural beauty and function in his *Dē Architectūrā*, while Frontinus's *Dē Aquaeductū* features water-flow calculations.

Philosophical motivations behind Ancient Greek mathematics did introduce a major limitation, the apparent incompatibility of geometry and arithmetic, due to the fact that the existence of irrational (or, more accurately, "incommensurable") proportions in geometry seemed to contradict the understanding of numbers at the time. Mathematicians such as Eudoxus simply suggested that arithmetic and its integral numbers should be kept separate from geometry and its possibly irrational magnitudes, leading to the need for geometrical representations of future mathematical innovations such as algebra. Still, arithmetic continued to develop independently among the more practical occupations of artisans and merchants. In fact, a mercantile textbook by Fibonacci, the *Liber Abacī*, helped contribute to the popularization of Hindu-Arabic numerals in Europe (Kline, 1990). Arithmetic was even designated one of the four subjects to master for the study of philosophy in the *quadrivium* outlined by the philosopher Boethius in the 6th century CE, alongside geometry, astronomy, and music.

Although the Middle Ages are not represented by the excerpts in this reader, we do not mean to suggest that the period was scientifically or mathematically barren. The notion of an intellectual "Dark Age" following the fall of the Roman Empire is now largely rejected by historians. Nor did the spread of Christianity militate against the progress of science, to cite another mistaken view of the period. The Catholic Church may have drawn attention away from what they labeled as pagan studies such as natural philosophy,[3] but the chaos following the collapse of the Western Roman Empire was likely more responsible for the loss of important works in Western Europe. Indeed, science held on well within the (also Christian) Byzantine Empire,[4] and was also embraced in the caliphates during the Golden Age of Islam. Moreover, through the granting of some of the first university charters, the Catholic Church helped establish the university as a protective and collaborative institution for scholars across Christian Europe and set the stage for the reintroduction of philosophical works — particularly Aristotle's — from the Islamic world via Spain. It is in these late medieval *scholae* that Scholasticism emerged, an inherently dialectical and instructional tradition often associated with the theologian Thomas Aquinas, in which scholars attempted to resolve the conflicts between the arguments of authorities ranging from Aristotle to the Bible in order to uncover a unified truth suffusing them all (Ostler, 2007). Contrary to popular belief, the tenets of the Aristotelian worldview were not held as dogmatic truths in the Middle Ages, even if the scientific language used remained largely Aristotelian. John Buridan's theory of impetus (a precursor to the modern notion of momentum) or Nicholas Oresme's questioning of certain cosmological principles (e.g. the

[3] Along these lines, some authors even allege that interest in astronomy was preserved primarily because of the field's relevance in determining the date of Easter (Gázquez, 2016).
[4] Lazaris (2020) provides an extensive overview of Byzantine science.

rotational immobility of the Earth) can be cited as examples (Grant, 1996).

In the latter half of the Renaissance, this state of science underwent a significant transformation that, as should be expected in a scientific discipline, did not occur without resistance or disagreements. This reader includes excerpts from works by three of the major participants in this movement, Copernicus, Kepler, and Newton. In short, the *Scientific* Revolution sprouted from a debate on *planetary* revolutions, with Copernicus devising and perfecting a heliocentric model in order to address what he perceived as theoretical inadequacies in previous geocentric systems. Kepler and Newton would then supply an explanatory basis for the counterintuitive idea of a moving Earth, with insight from Galileo's investigations into the nature of movement.[5] As the grandeur of its name would suggest, the Scientific Revolution is the epitome of a paradigm shift (as defined by the 20th-century philosopher of science Thomas Kuhn), where one scientific worldview, with all its foundational beliefs and analytical approaches, supersedes another. One major change was, of course, the elevated role of the laboratory experiment, although it was not the exclusive method of deduction. Indeed, Newton's excerpt in this reader describes the cannonball thought experiment that he used to supplement his more formal mathematical reasoning, and this in turn was preceded by Galileo's combination of physical and thought experiments in his dismantling of Aristotelianism.

[5]Galileo is also noteworthy for having shown that a vernacular language can be fit for communicating these groundbreaking mechanical results, although subsequent translations from his Italian into Latin helped spread his works across the scientific community (Oosterhoff, 2014).

Another major consequence of this revolution was the rejection of the conceptual division between the terrestrial and celestial domains (a rejection precipitated by Tycho Brahe through his meticulous monitoring of an eponymous supernova [Mark, 2018]) and the understanding that the Earth behaves according to the same natural laws as other celestial bodies. With this unification came the realization that the mathematical techniques used in astronomy could be applied to other areas of study under common theoretical frameworks, such as forces in the Newtonian science or corpuscles in the Cartesian mechanistic philosophy. No longer feeling tied to Aristotelian models, scientists also gradually recognized the diversity of scientific subjects and the potential need for a unique kind of methodology outside of physics. As elaborated in her introduction in this reader, Merian is arguably a forerunner for this trend through her fieldwork and illustration practices, especially at a time when biology rarely strayed far from comprehensive natural philosophies. More information on the classification of species, which would foreshadow Darwinian evolution, is provided in the next section in the context of Latin as a language of science.[6]

Changes were also occurring at around the same time within mathematics itself via the coalescence of geometry and algebra thanks to the introduction of the Cartesian plane in *La Géométrie*[7] by René Descartes, who built upon the geometric toolbox for algebra developed by Al-Khwarizmi and other predecessors (Oosterhoff, 2014). As mathematics became more interconnected, it also

[6]See DeWitt (2018) for a more detailed introduction to the main stages of the Scientific Revolution and relevant concepts in the philosophy of science.

[7]Strictly speaking, it first appeared across the four editions of the more extensive (and more widely-read) Latin translation. In the French original, Descartes made do without a definite perpendicular y-axis or ordinate (Burton, 2011).

split into areas (beyond simply arithmetic or geometry) that each required a unique approach or set of base theorems. Examples include calculus in the 17th century or number theory, finally recognized as a field independent from arithmetic in the 19th century through the contributions of Gauss, which are incorporated into this reader. This expansion in turn demanded more abstract formulations and a reconsideration of the foundations for mathematical truth. Indeed, mathematics experienced its own sort of paradigm shift at the beginning of the 19th century when it was confirmed that the axioms underlying the Euclidean geometry, which had remained largely untouched for millennia, were not necessary but could be altered to produce new classes of geometries (Dahan-Dalmedico and Peiffer, 1986). This period of intensive change in mathematics extended past the turn of the 20th century, particularly in the study of axiomatics, as touched upon in the introduction to Peano. In contrast, following the establishment of James Clerk Maxwell's electromagnetic theory and Ludwig Boltzmann's statistical thermodynamics, many physicists regarded their field as mostly complete by the end of the century. Yet, as remarked by Lord Kelvin in 1900, two "very dense" clouds were already starting to brew that would turn out to foreshadow the revolutions brought by special relativity and quantum mechanics. However, their development lies beyond the period of Latin as the language of science, the era of relevance for this reader.

3 A Genealogy of the Language of Science

To complement this rudimentary history, we also trace the evolution and influence of scientific Latin over nearly two millennia, from the 1st century BCE to the 19th century CE. Over this time, Latin

became revered for the longevity and universality that it had acquired through church and empire, so that, even after its "death," it was frequently viewed as an ideal language for international communication (a recurring attitude across many of the excerpts of this reader) in much the same way the Roman Empire would come to serve as a model or baseline for transcontinental empires in the medieval and modern ages. The most obvious proof of the legacy of Latin is its omnipresence across the scientific registers of most modern European languages, including English, which has become at present a sort of successor to Latin as the *lingua franca* of Western academia.

However, to assume this position in the first place, Latin had to overcome a (at the time) much more cemented language for science in both popularity and useability, Ancient Greek. The historical importance of Ancient Greek should come as no surprise to scientists, who observe Greek roots in the names of the main subdivisions of science (physics, chemistry, biology, etc.) or of the basic chemical elements.[8] Since the Ancient Greeks developed the advanced portions of their philosophy fairly independently from other traditions, they were able to evolve their thinking in accordance with the advantages and limitations of their language and invent new terminology following its syntactic rules if needed. Some philosophically useful grammatical features of Greek include its more extensive set of participles when compared to Latin, or its many constructions involving definite and indefinite articles (corresponding to the English "the" and "a/an"): On the one hand, the indefinite article in isolation can refer to a given class of "things" at hand in a text of philosophy,

[8]Technically, names such as "hydrogen" or "oxygen" are French coinages constructed from actual Greek borrowings. After all, modern chemistry is usually traced back to Antoine Lavoisier, who wrote primarily in French (Korenjak, 2023).

without the need for disruptive specifications; on the other hand, the definite article permits any adjective or verb to be transformed into a noun encapsulating an abstract state of being, quality, or process (Powell, 1999). Thanks to the Greek language's long connection to erudition or simply out of convenience, Roman thinkers with varied backgrounds preferred writing in Greek. They include Roman citizens who were native speakers of Greek (e.g. the astronomer Ptolemy, the historian Plutarch, and even the physician Galen, who practiced in Rome) and Roman elites who were tutored in Greek as an expected second language (from the historian Quintus Fabius Pictor to the philosopher-emperor Marcus Aurelius). More generally, Greek persisted as the language for the inhabitants and administrators of the eastern portion of the Roman Empire (Ostler, 2007).

Notably, Ancient Greek had a more significant impact on the development of Latin as a technical language in oratory, grammar, and philosophy than the other way around (except in law) (Roelli, 2021). Here it should be noted that the theoretical component of Greek science prioritized in our timeline above was not able to rival the appeal of practical applications such as medicine or architecture among the Romans, in which they were extremely skilled. The Romans, the patriotic Cicero included, adopted and transliterated many specialized Greek terms such as *philosophia* freely or even inserted entire Greek phrases in their personal writings. However, people intending to publish philosophy in Latin, especially this reader's two representatives of the classical era, Cicero and Lucretius, often had external motivations for their choice of language and their adherence to purisms, and reflected directly on the challenges they experienced as a result. Cicero believed that Latin had just as much as potential as Greek to become an analytical language, with him referencing ex-

amples of Latin words possessing greater specificity than their Greek counterparts, e.g. *vultus*, *labor*, and *dolor* (Powell, 1999). As an orator, he valued clarity (Roelli, 2021) and practiced paraphrastic translation, with a frequent use of near-synonyms to either narrow in on the desired meaning, account for a Greek word's generality (Powell, 1999), or simply explicate an author's intent (Hendrickson, 2020). Cicero is also notable for his Latin neologisms paralleling existing Greek words in surface etymology, the examples most recognizable to a modern reader being *quālitās* for ποιότης (lit. "how-much-ness"), *essentia* for οὐσία ("be-ing"), or *comprehēnsibilis* for καταληπτός ("graspable," from a Stoic analogy between the mind and a closing fist). Lucretius employed similar practices in his *Dē Rērum Nātūrā* within the context of poetic conventions: For instance, a set of Latin analogues to a single Greek word could be tied together with visual metaphors to more fully capture the original concept, while providing the literary flourishes demanded in poetry. In this way, the poet worked around the self-perceived "poverty of [his] language" (*egestātem linguae*) within the boundaries of the Latin epic genre, which has been interpreted as an effort by Lucretius to distance the philosophy he wants to defend from its Greek origins (Sedley, 1999).

While Classical authors could presume a familiarity with Greek among their educated audience if they needed to borrow new words, this basic knowledge of Greek and hence Greek thinking was gradually lost in Late Antiquity. The philosopher Boethius and the archbishop Isidore of Seville attempted to counteract this loss of knowledge by penning encyclopedic works in Latin. Their writing styles seem almost opposite in approach: Boethius favored literal translation into Latin to keep the logical structure of the original works intact, while Isidore famously utilized simple grammar in his exten-

sive *Etymologiae*, albeit with some technical or vernacular vocabulary used throughout (Roelli, 2021). As discussed earlier, towards the middle of the Middle Ages, Arabic treatises entered Europe and encouraged a new round of intensive translation into Latin, with scholars such as Adelard of Bath resorting to transliterations and calques from Arabic much more liberally than Cicero or Lucretius would maybe have liked (Ostler, 2007). Many of these loanwords, e.g. algebra, algorithm, alchemy, and alcohol, are recognizable due to the prefix *al-*, the Arabic article, which arguably carries just as much versatility as its Greek counterpart (for example, it can be appended to participles to describe actions in the abstract, as can be done in contemporary English). Scholastic thinkers took advantage of grammatical features that had become more widespread in Late and Medieval Latin in order to break up and organize their premises, logical relations, and descriptions of natural processes: the use of *quod* with an indicative to clearly demarcate an indirect statement (as opposed to a more formal accusative-infinitive construction); the increased frequency of prepositions to distinguish between case usages (e.g. *per* for what would in Classical Latin commonly be an ablative of means); and an extension of the passive to express the reflexive middle voice (e.g. "he raises himself") or any action without a clear agent.[9] Moreover, the Scholastics actively invented new words to accommodate constructions previously exclusive to Greek and Arabic. Indeed, they adopted *ēns*, the present participle ending of verbs, as a makeshift present participle for *esse* to capture the idea of a "being/thing" (the analogue in Greek, the participle ending ὄν, functioned as not just a noun but also a participle). Similarly, *ly* (from the French *le*) was chosen as a Latin article to introduce a

[9]A useful piece of advice for a student memorizing deponent verbs is that they tend to have an implicit reflexive, middle sense. Classical poetry also often employed the passive for the middle voice in imitation of Greek.

word or phrase as an object of study in itself (Ostler, 2007).[10] Note that, during the late medieval period, when the vernacular languages gradually matured through specialized orthography and even their own literary genres and traditions (e.g. the chivalric romance of the Romance languages), people also started to communicate ancient scientific thinking using their native languages.[11]

In another important transition, the Renaissance would trigger, not necessarily a "rebirth," but a "reset" for the Latin language in the form of humanism, an intellectual movement in part sparked by the arrival of Greek scholars from the collapsing Byzantine Empire. The resulting reintroduction of primary source Greek manuscripts that were thought to have been lost (notably Plato's corpus) helped reinstate Greek as a key language for reading. Through an increased understanding of classical Roman works, the humanists also cemented ancient Latin, particularly Ciceronian Latin and its flowery and persuasive oratory, as the Latin style to be imitated in writing (Ostler, 2007). In contrast, they denounced the dry rigor of the Scholastics, their supposed interest in meaningless questions ("How many angels can stand on a pinhead?"), and their deviations from Classical Latin, mocking their blatant violation of perceived Latin rules in the construction of vocabulary. Although leading scientists emerging from this period of humanism may have espoused some of these views, in practice they upheld many features of Scholastic Latin that were conducive to their purposes (Roelli, 2021). Moreover, the spread of humanism was non-uniform: At its height in Italy during the early 16th century, humanism was only gradually gaining a foothold in

[10] See Mantello and Rigg (1996) for a fairly comprehensive treatment of Medieval Latin vocabulary and grammar.
[11] Goyens et al. (2008) contains a variety of chapters on individual medieval translations into vernacular languages like English or French.

northern European institutions, possibly explaining the presence of what could be considered scholastic literary habits in Copernicus's earlier writings that would become less conspicuous in his major publication, the *Dē Revolūtiōnibus* (Borski and Kokowski, 2021).

Humanism did have its own fair share of positive consequences on the development of science. Not only did scientists and humanists routinely engage with each others' fields (Blair, 1999), with Latin eloquence taught alongside the classical *quadrivium*, but the humanist and scientific approaches to translation were frequently seen as mutually beneficial, with the former paying attention to the proper phrasing of definitions and the latter detecting mistakes in lines of reasoning (Oosterhoff, 2014). The diversification of the pool of classical sources of authority through the efforts of humanists may have also made scholars in the Scientific Revolution more comfortable with the idea of challenging Aristotle (Cochrane, 1976), as others in antiquity had done, and as Copernicus would do in the excerpt included in this reader.

While the Latin following the rise of humanism, now usually labeled as Neo-Latin, exhibits few major grammatical differences from Classical Latin, every individual naturally developed their own unique style, likely correlated with their area of study: As a pair of examples drawn from the authors in this reader, Leibniz's prose has been classified as leaning more towards scholasticism, which should be expected given his inquiries into the philosophy of being, whereas the style of Newton's *Prīncipia Mathēmatica*, full of propositions and diagrams, was influenced by the concision found in translations of Euclid (Roelli, 2021). More generally, just like modern scientific English, scientific Latin often follows a straightforward sentence struc-

ture, especially thanks to textual innovations such as section divisions and punctuation (Korenjak, 2023). An insightful primary-source case study is a letter by Gauss, who published and even wrote diary entries in Latin and was an eager emulator of Classical Latin: Reflecting on the overuse of what would in Classical Latin be described as future-less-vivid conditional constructions, he admits "that *sī* with the subjunctive is not good Latin; but modern mathematicians seem to have made for themselves the rule of constantly using the subjunctive in hypotheses and definitions."[12] Indeed, the subjunctive is quite common in any situation where hypotheticality and, by extension, generality is implied; hence, *sit* sets up the definition of variables in much the same way as the English "let." In our excerpts, readers familiar with modern scientific and mathematical writing will also immediately recognize the use of the impersonal passive or the first-person plural (*plūrālis modestiae*) in cases where the agent is irrelevant, such as in proofs.

Likewise, after the Renaissance, Latin terms in physics and mathematics often retain meanings close to or analogous with their Classical or Late Latin originals (Korenjak, 2023), as exemplified by Leibniz's *differentiālis*, Bernoulli's *integrālis*, and even Newton's counterparts of *flūxiō* and *fluēns*. (To see firsthand the creation of such terminology pulled from Classical Latin, consult Gauss's definition of modular arithmetic in this reader.) Some interesting exceptions include *logarithmus*, which the mathematician John Napier constructed from the Greek λόγος, "ratio," and ἀρίθμος, "number," due to the function's role in reducing division to subtraction; or *spectrum*, previously used by Cicero as a calque of the

[12] This translation of Gauss's letter is taken from Dunnington (1955).

Greek εἴδωλον (Sedley, 1999) and ultimately revitalized by Newton in English in his studies on light.

Areas devoted to large-scale classification, whether of substances in chemistry or species in biology, were much more active in this expansion of vocabulary, from a desire for international comprehensibility and consistency (Berrens, 2025), which are essential to the scientific principles of reproducibility and verifiability.[13] Biologists had to tackle an ever-increasing class of newly discovered species, especially in the New World (e.g. our excerpt of Merian features *Guaiāva* for "guava" and *Colobritgens* for "hummingbird," both derived from Native American languages), or needed to revisit known species possessing a vernacular name, but no Latin equivalent (e.g. Leonhart Fuchs repurposed the classical *Digitālis* to mimic the German name for a foxglove, "Fingerhut" [Berrens, 2025]). In the 18th century, Carl Linnaeus would finally formulate a consistent naming scheme for all species, grounded in Latin — his system in botany even incorporated a dash of Latin grammar via a brief ablative of description (Ostler, 2007). Just as scientists writing in Latin were open to inserting Greek phrases whenever appropriate (as seen in this reader's selections for Kepler and Leibniz), Linnaeus readily adopted, transliterated, and combined Greek words in his scientific names, although he insisted that they should not depend on vernacular languages (Berrens, 2025). Latin holds on to the present day in taxonomy and binomial nomenclature as a shared reference point for all biologists.

[13]Indeed, in the aftermath of humanism, early modern naturalists often referenced as a sort of cautionary tale the many debates regarding the identity of species described by ancient authors to justify a new and, hopefully, more reliable terminology (Berrens, 2025).

Finally, it should be noted that, in the history of science and philosophy, there were many attempts at developing a universal language adapted to not just scientific communication, but the process of reasoning itself — for more information, see the introductions to Leibniz and Peano's excerpts in this reader. These visions did not quite materialize, and Latin itself declined in usage by the 19th century. Indeed, throughout the modern period, the vernacular languages accrued greater authority in scientific matters, and nearly every figure in this reader published in at least one language other than Latin: Leibniz wrote in French and German (contributing to the latter's role as a language of thought [Roelli, 2021]), Merian in German and Dutch, Newton in English, Euler in French and German, Gauss in German and occasionally French, and Peano in Italian. With some exceptions like German, much of the vernacular technical vocabulary was directly taken and morphologically standardized from Latin, with some translations of Latin works even providing glossaries furnished with explicit definitions for the effectively untranslated Latin borrowings (Waquet, 2023). Gauss's *Disquīsītiōnēs Arithmēticae* from 1801 is often cited as the last significant mathematical treatise written in Latin (Ostler, 2007), but this factoid overlooks the continual influence of Latin in the minds of the members of the scientific and mathematical community for the rest of the century, even beyond the tradition of assigning Latin titles to non-Latin works. As late as 1861, the German mathematician Bernhard Riemann wrote his entry for a prize category created by the French Academy of Sciences in Latin to compensate for his inexperience in French. Students may relate to the fact that Riemann submitted his paper right at the deadline. While this paper did not win the prize, it would nevertheless pave the way for his contribu-

tions to differential geometry, a core element of special and general relativity (Farwell and Knee, 1990).

4 About this Edition

The corpus of Cicero and the epic poem by Lucretius have been under the purview of Classical scholars for centuries and have received a fair share of critical editions and republications. Accordingly, we have decided to base our text for the *Tusculānae Disputātiōnēs*, *Dē Nātūrā Deōrum*, and *Dē Rērum Nātūrā* on existing critical editions, namely the Teubner editions for Cicero and W. E. Leonard and S. B. Smith's edition for Lucretius. We modify punctuation and capitalization, which the authors themselves would not have employed, whenever we believe that the change would help clarify sentence structure to a user of this reader. We base most of the post-Classical works on their first published editions that are provided by the Internet Archive or Google Books. The only notable exception is Copernicus, whose *Commentāriolus* has already been transcribed in the fourth volume of the *Nicolaus Copernicus Gesamtausgabe*. The work also provides a collation of variants across the three known manuscript copies of the *Commentāriolus*, which are available in the fourth volume of *Nicolaus Copernicus Complete Works*. (All works are cited in full in the author's respective chapter.) We have preserved most of the punctuation present in these early modern texts, unless doing so would lead to unnecessary confusion. A Neo-Latin punctuation convention adopted by Kepler in his *Somnium* merits some explanation: The modern usage of colons and semicolons is a narrowing of their original purpose to indicate varying lengths of pauses in sentences alongside commas and periods; the hierarchy consists of commas for the shortest pauses (where conjunctions are

still required to join independent clauses), semicolons, colons, and periods for the longest. With this background information in mind, a modern reader can more easily follow this reader's excerpt for Kepler and is further assisted by the removal of a couple of commas in seemingly unnatural and interruptive locations. Any differences in readings from our source texts are listed at the end of each author introduction, with the reading in the source text on the left and our reading on the right.

Another orthographical decision that needs to be briefly justified is the addition of macrons. Their pedagogical role is to help students read Latin aloud and internalize its vocabulary like a living language, following its reconstructed ancient pronunciation, and hence macrons are appropriate in Classical works even if they are a post-Classical invention like punctuation. Moreover, in the case of Lucretius, macrons aid in scansion by indicating which vowels are long by nature. In contrast, the standard pronunciation of early modern Latin was not only non-Classical, but dependent on the nation of the speaker. Nevertheless, the humanists, with their mission of restoring Classical styles of writing, would likely have encouraged the use of macrons had they been aware of ancient pronunciation conventions. Finally, Peano's *Latīnō Sine Flexiōne* may not be actual Latin, but he envisioned it as both a written and spoken language, actively recommending specific pronunciation for certain sounds (primarily when they deviate from the standards of his mother tongue, Italian). Peano makes no note of vowel length, but we include macrons as consistent with his belief that "the better model" for pronunciation "is the ancient one" (*Fōrma meliōra es antīquō*).[14] Most importantly for a student reader, macrons are also practical for distinguishing easily

[14] See the section titled *§ 7. – Prōnūntiā dē Latīnō.* in Peano (1903).

misidentified inflections (e.g. the feminine ablative from the nominative). The source texts for Copernicus, Kepler, Leibniz, Merian, and Newton make use of a Neo-Latin solution to this problem, the accent system, where acute, grave, and circumflex accents help distinguish between words that are written identically but represent different parts of speech or different cases. Only rarely do the accents have bearing on pronunciation. We have replaced these diacritical marks with macrons for the sake of consistency. Note that we add macrons to post-Classical vocabulary only where predictable, e.g. in known prefixes or suffixes, or inferable from a word's Greek or Arabic roots.

We also provide on-page vocabulary for the convenience of students, so that they do not have to interrupt their reading to consult a dictionary that, due to this reader's focus on Neo-Latin works, may not even contain the word they are searching for! They can then allocate that time to reading more Latin, deepening their understanding of the language and supporting their growth as Latin scholars and modern participants in this international and intergenerational scientific conversation. To avoid clutter, we relegate the most common Latin words to a glossary at the end of the reader, as a student more likely than not already knows their meaning. Any specialized or non-Classical definitions of basic words are still supplied in the on-page vocabulary. Although we have used the DCC Latin Core Vocabulary as a point of reference, we have not defined a specific threshold for inclusion in the back glossary, because the DCC list is not tailored to the vocabulary of scientific literature. The vocabulary words are also arranged in the order that they appear in the text.

Our on-page commentary helps to dissect potentially confusing Latin phrases or clauses. Furthermore, since our target audience may not be familiar with an author's references to other texts or the scientific or historical context behind a certain statement, we have made sure to provide this line-specific information in the commentary while heeding space restrictions. Sometimes this task is undertaken by the infoboxes when short commentary notes may not be sufficient.

Finally, we note some miscellaneous editorial decisions: Whenever we have to split particularly lengthy sentences across two pages or exclude text in between excerpts, we clearly indicate this division or omission. For the most part, we also keep the spelling conventions of our authors and their time periods, equating them with their Classical Latin counterparts in the vocabulary (e.g. *quum* for *cum*, only when used as a conjunction). The main exception is the conversion of j's into consonantal i's, the odd vocalic v into u, and the long 's into simply s's.

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Abbreviations

abl.: ablative
acc.: accusative
adj.: adjective
adv.: adverb
cf.: compare to (cōnfer)
dat.: dative
f.: feminine
gen.: genitive
impers.: impersonal
impf.: imperfect
indecl.: indeclinable
indic.: indicative
inf.: infinitive
lit.: literally
m.: masculine
PAP: present active participle
pass.: passive
pl.: plural
PPP: perfect passive participle
prep.: preposition
⇒**:** continued on next page

Chapter 1 — Cicero: *Tusculānae Disputātiōnēs* & *Dē Nātūrā Deōrum*

Marcus Tullius Cicero (106 BCE – 43 BCE) may seem to be a surprising choice for the first author in this reader. After all, he is best known, not as a scientist, but as an eloquent orator and consul who tried to "preserve" the Roman Republic in its final years. Although Cicero doesn't have a scientific or mathematical text under his name (excerpt perhaps a portion of his *In Verrem* speeches exposing the corruption of the governor Gaius Verres, in which he uses detailed financial calculations to project an air of rigor and certainty [Cioffi, 2011]), he did translate some early scientific Greek works,[1] in addition to authoring multiple philosophical treatises of his own. Indeed, Cicero adhered to the school of Academic Skepticism, which he encountered in the works of the 2nd-century BCE Greek philosopher Clitomachus: Skeptics believed that logic cannot prove any statement to be absolutely true or false ("the problem of the criterion"), thereby forcing them to assume ἐποχή or a "suspension of judgment." In his works, Cicero criticized other schools for accepting their main tenets without scrutiny and he instead tried to carefully judge their persuasiveness (but not necessarily their verity) (Vogt, 2022). The role (and intensity) of skepticism may have evolved over the millennia, but it still forms an essential element of modern science.

As an example of the Academic Skeptic's approach, in Book V of the *Tusculānae Disputātiōnēs*, Cicero evaluates and ultimately defends the Stoic principle that virtue is sufficient for happiness, notably by comparing the lives of historical figures in pairs. The turpid tyrant

[1] Among Cicero's translations are Aratus's *Phaenomena*, a non-technical poem on astronomical and meteorological phenomena, and at least part of Plato's *Timaeus*, a philosophical dialogue on the creation of the world.

Dionysius I of Syracuse and the modest mathematician Archimedes are one such pair of *exempla*. Before exalting his restoration of Archimedes's tomb, the focus of the first excerpt, Cicero first discusses the life of Dionysius in part in the context of the well-known parable of the Sword of Damocles: Damocles believed that the life of a king was happy and idyllic, but in order to prove the contrary, Dionysius offered to make him king for a day; he set up a sword hanging above the throne by a thin thread to represent the constant dangers of plots and betrayals from the enemies Dionysius had made during his tyrannical rise to power. Despite describing Dionysius's life in great detail for ten pages, Cicero hardly devotes a page to his moral counterpart, the mathematician Archimedes from the same city (Jaeger, 2002). Since Cicero glosses over the life of the subject of his appraisals, let us also briefly mention some of Archimedes's achievements.

In geometry, Archimedes is credited with the method of exhaustion, a way to calculate π by approximating the circumference of a circle with the perimeter of polygons with an increasing number of sides. He also determined the volume and surface area of a sphere (see the commentary below for the proof) and the area contained within a parabola and a straight line. In physics, he first stated the law of the lever and the principle of buoyancy named after him, which he is said to have discovered while in a bath before exclaiming, εὕρηκα, "I have found (it)," and running around Syracuse naked (Biello, 2006).

The circumstances of Archimedes's death, relevant to this excerpt, also form an interesting connection with the Second Punic War. In 213 BCE, the praetor Claudius Marcellus led his army to

Syracuse, which had allied itself with Carthage two years earlier, and besieged the city. After facing the many legendary battle machines created by Archimedes, including a claw that could allegedly capsize warships from afar and a "heat ray" that used parabolic mirrors to concentrate light beams towards attacking ships (Rorres, 2017), the Romans finally captured and stormed Syracuse in search of spoils.

Marcellus likely held Archimedes in high regard due to his ingenious inventions and explicitly forbade his soldiers from killing him, although he did want them to capture him (presumably so that he could design machines for the Romans). According to one account of the mathematician's death (by the historian Valerius Maximus), when soldiers busting into his house to loot it asked him who he was, he responded, "I beseech you, do not disturb that (sand)" (*'Nōlī, . . . obsecrō, istum (pulverem) disturbāre,' Factōrum et Dictōrum Memorābilium Librī Novem* VIII.7.ext.7) in which he was drawing some geometrical figures, implying that he was more concerned with his studies than his own survival. As the story goes, a soldier immediately stabbed him and the blood soaked into the sand. It is said that the praetor was greatly offended and buried Archimedes in a tomb immortalizing his favorite result, his calculation of the surface area of a sphere (as corroborated by Cicero's excerpt). It is not known whether this anecdote from Valerius Maximus is true, but it does speak to how mathematicians such as Archimedes were perceived by the Romans. Of course, Cicero himself acknowledged the Romans' preference for the practical applications of mathematics, from architecture and aqueduct infrastructure to surveying and chronometry.[2] This sentiment is humorously echoed by the 20th-

[2]While comparing the intellectual interests of the Greeks and Romans, Cicero writes that "geometry was in the greatest esteem among those [Greeks], and so nothing [was] more distinguished than mathematics; but we have limited the utility of this art to measuring and

century mathematician George F. Simmons' assertion that Cicero's "act of respect in cleaning up Archimedes' grave was perhaps the most memorable contribution of any Roman to the history of mathematics" (Simmons, 1992).

The second, more analytical, excerpt comes from Cicero's *Dē Nātūrā Deōrum*. As its title suggests, the book investigates whether the existence of gods can be proven and their nature determined through reason alone, which was a naturally controversial issue in Greek philosophy: The sophist Protagoras adopted the agnostic perspective, believing that the gods are beyond the limits of human comprehension; Zeno of Citium (see the commentary below) argued that the fact that nearly every civilization prays to some collection of gods implies that there must be beings to warrant such devotion; meanwhile, atheists objected that religion was merely a tool of social control, designed to keep people in line under the fear of divine punishment (as was once referenced by the Greek playwright Critias) (Balme et al., 2016).

The *Dē Nātūrā Deōrum* is a fictional dialogue between three of Cicero's contemporaries defending their own school of thought in one book each: the Epicurean Gaius Velleius (who believes in happy gods that do not care about humans), the Stoic Quintus Lucilius Balbus (who interprets the regularity and usefulness of the world as evidence for the existence of gods), and Gaius Aurelius Cotta (who simply contests the others' claims and, as a skeptic, likely speaks for Cicero). In the excerpt, Cotta refutes Balbus's argument that, since a thing with reason is better than one without and the universe is

calculating" (*In summō apud illōs honōre geōmetria fuit, itaque nihil mathēmaticīs inlūstrius; at nōs mētiendī ratiōcinandīque ūtilitāte huius artis termināvimus modum*, *Tusc. Disp.* I.II.5).

better than anything it contains, the universe must possess reason and hence must be god itself.

Source Texts: Our Latin texts come from Heine (1881) and Goethe (1887), with a few changes in spelling and one deviation in reading.

Pages 45 and 48	*sepulchrum, sepulchrō, sepulchrōrum*	*sepulcrum, sepulcrō, sepulcrōrum*
Page 48	*Agragentīnās*	*Agragantīnās*
Page 48	*Inmissī*	*Immissī*
Page 49	*Adpārēbat*	*Appārēbat*
Page 49	*dīmidiātīs*	*dīmidiātum*
Page 51	*adsentior*	*assentior*
Page 52	*pulcherrumae*	*pulcherrimae*

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Tusculānārum Disputātiōnum Liber Quīntus

XXIII. 64. Nōn ego iam cum huius vītā, quā taetrius, miserius, dētestābilius excōgitāre nihil possum, Platōnis aut Archȳtae vītam comparābō, doctōrum hominum et plānē sapientium:

Tusculānus, -a, -um: of Tusculum, Tusculan
disputātiō, -ōnis f.: debate, disputation
quīntus, -a, -um: fifth
taeter, -tra, -trum: horrid, disgraceful
dētestābilis, -e: abominable, detestable
excōgitō (1): think of, contrive of
Platō, -ōnis m.: Plato (Latinization of Πλάτων; see below)
Archȳtās, -ae m.: Archytas (Latinization of Ἀρχύτας; see below)
comparō (1): (here, with *cum*) compare (something) with (another thing)
doctus, -a, -um: learned (lit. "taught," from *doceō, -ēre, -uī, -tus*)
plānē (adv.): clearly
sapiēns, -entis: wise

---

cum huius vītā: take closely with *Platōnis . . . comparābō*; *huius* refers to Dionysius I of Syracuse

quā taetrius, miserius, dētestābilius: this swift series of comparative adjectives, which modify *nihil*, is an instance of asyndeton; *quā* (which has the antecedent *vītā*) is an ablative of comparison

Platōnis aut Archȳtae vītam: Plato is of course the famous Greek philosopher, the student of Socrates and the teacher of Aristotle; Archytas was his friend and a mathematician in the school of Pythagoras (which he in fact directed after Pythagoras's death)

doctōrum . . . sapientium: genitive plurals in apposition to *Platōnis* and *Archȳtae* simultaneously

ex eādem urbe humilem homunculum ā pulvere et radiō excitābō, quī multīs annīs post fuit, Archimēdem.

---

humilis, -e: humble
homunculus, -ī m.: little man, weak man
pulvis, pulveris m.: dust, sand (in which Archimedes would have sketched geometrical diagrams)
radius, -iī m.: rod (used both for drawings in the sand and for measurement)
excitō (1): summon forth
sum, esse, fuī, futūrus: (here) live
Archimēdēs, -is m.: Archimedes (Latinization of Ἀρχιμήδης)

---

ex eādem urbe: *from the same city* as Dionysius; considering that all the pairs of figures in his prior moral exempla shared some similarities (e.g. the pairing of two aspiring consuls), it is likely not a coincidence that he chooses to compare two historical figures from Syracuse

humilem: this quality is quite important for Cicero's analysis: after recounting this anecdote, he asks the reader whether they would prefer to live Dionysius's life of power and fear, or Archimedes's life in a humble station (indeed, his grave was entirely forgotten), but one enriched by a passion in the sciences and mathematics ("the sweetest food of minds," *suāvissimus pāstus animōrum*, *Tusc. Disp.* V.XXIII.66)

quī multīs annīs post fuit: *who lived many years later* (a century later, to be exact, as Dionysius lived in the 4th century BCE); the antecedent of *quī* is *homunculum*, to which *Archimēdem* is in apposition; *multīs annīs* is an ablative of degree of difference

Cuius ego quaestor ignōrātum ab Syrācūsānīs, cum esse omnīnō negārent, saeptum undique et vestītum vepribus et dūmētīs indāgāvī sepulcrum. Tenēbam enim quōsdam sēnāriolōs, quōs in eius monumentō esse īnscrīptōs accēperam, quī dēclārābant in summō sepulcrō sphaeram esse positam cum cylindrō.

---

quaestor, -ōris m.: quaestor (the lowest office of the *cursus honōrum*; Cicero became quaestor of Sicily in 76 BCE at the lowest eligible age, *suō annō*)

ignōrō (1): not know

Syrācūsānus, -a, -um: Syracusan (including as a substantive)

omnīnō (adv.): altogether, at all

negō (1): deny

saepiō, -īre, saepsī, saeptus: surround, enclose; cover

undique (adv.): from all sides; completely

vestiō, -īre, -īvī, -ītus: cover, surround

veprēs, -is m.: bramble-bush

dūmētum, -ī n.: thicket

indāgō (1): track, search for, hunt for

sepulcrum, -ī n.: grave, tomb

teneō, -ēre, -uī, -tus: (here) hold in one's mind, remember

sēnāriolus, -ī m.: little verse (consisting specifically of six feet; diminutive of *sēnārius, -iī m.*)

monumentum, -ī n.: monument

īnscrībō, -ere, -scrīpsī, -scrīptus: inscribe

accipiō, -ere, -cēpī, -ceptus: (here) learn

dēclārō (1): declare

sphaera, -ae f.: sphere (from the Greek σφαῖρα)

cylindrus, -ī m.: cylinder (from the Greek κύλινδρος)

Cuius: genitive of possession with *sepulcrum*; a relative pronoun can be translated as a demonstrative pronoun

cum esse omnīnō negārent: *since they denied (it) existed at all*; a causal *cum* clause with an imperfect subjunctive verb; the subject of *negārent* is the Syracusans and the implicit subject of *esse* is *sepulcrum*

saeptum undique et vestītum vepribus et dūmētīs: *saeptum* and *vestītum* are PPPs modifying *sepulcrum*, just like *ignōrātum*; *vepribus* and *dūmētīs* are ablatives of means

quōs . . . quī: the antecedent for both relative clauses is *sēnāriolōs*

in summō sepulcrō: *at the top of the grave*

Archimedes's Sphere and Cylinder

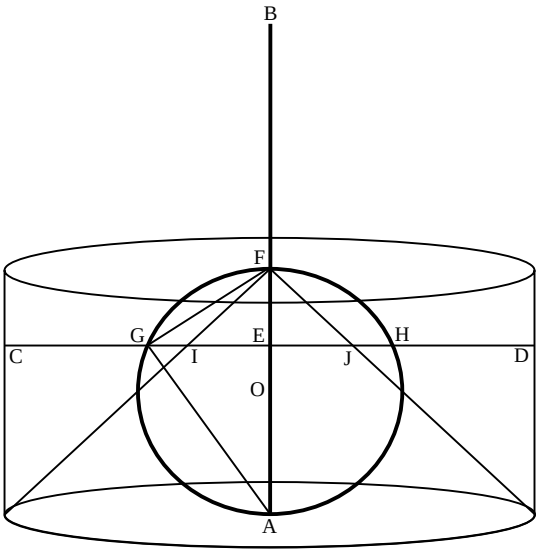


Figure 1: What is perhaps most interesting in Archimedes's derivation of the formula for the volume of a sphere (from *The Method of Mechanical Theorems*) is that, since formal integration did not exist yet, he had to employ an argument based on levers and centers of mass, physical analogues of an integral. Surprisingly, his proof requires a cylinder with double the width seen in most depictions of Archimedes's sphere-cylinder diagram.

Imagine a lever AB with length $4r$ and a fulcrum at the midpoint F. Let the sphere with radius r be centered at the midpoint O of AF. Then construct a cylinder with height and radius $2r$, with its bases at A and F, as well as an inscribed cone with the same dimensions. For any perpendicular CD to the lever AB, denote the intersection points with the sphere, cone, and lever by G and H, I and J, and E, respectively. Since EFG and AFG are similar right triangles, $EF : FG = FG : AF$, so $AF \cdot EF = FG^2 = EG^2 + EF^2 = EG^2 + EI^2$. $\Rightarrow$

Figure 1 (continued): Now consider the ratio $BF : EF$, which is clearly equal to $AF : EF$. By the previous result, $AF : EF = AF^2 : (AF \cdot EF) = CE^2 : (EG^2 + EI^2)$. Since the area of a circle is proportional to its radius squared, the ratio between the area of the cross-section of the cylinder (with radius CE) and the sum of the areas of the cross-sections of the sphere and cone (with radii EG and EI respectively) is equal to the ratio between BF and EF.

By the law of the lever, if those two cross-sections were moved to B, they would perfectly balance the cylinder's cross-section. If the sphere and cone are shifted such that their center of mass is at B (implying that their cross-sections can be collapsed onto that point), then their union would balance with the entire cylinder. Since the latter's center of mass is located at O, the ratio between the volumes of the sphere-cone combination and the cylinder is equal to the ratio of BF and OF, which is $2 : 1$. It is known that the ratio between the volumes of the cone and cylinder is $1 : 3$, so the sphere has one sixth of the volume of the cylinder, resulting in the familiar expression $\frac{1}{6} \cdot \pi(2r)^2 \cdot (2r) = \frac{4}{3}\pi r^3$.

Since the surface of a sphere can be dissected into infinitesimally small triangles, the sphere can also be represented as the composite of pyramids with a height of r and with the triangles as their bases. Since the volume of a pyramid is $\frac{1}{3}$ times the height times the base, the surface area of the sphere is ultimately $3 \cdot \frac{4}{3}\pi r^2 = 4\pi r^2$. Both theorems are proven more rigorously, albeit less straightforwardly and in the opposite order, in *Archimedes's On the Sphere and Cylinder*. This achievement is now commemorated on the back of the Fields Medal, the most prestigious award in mathematics (see front cover of this reader).

65. Ego autem, cum omnia conlūstrārem oculīs (est enim ad portās Agragantīnās magna frequentia sepulcrōrum), animum advertī columellam nōn multum ē dūmīs ēminentem, in quā inerat sphaerae figūra et cylindrī. Atque ego statim Syrācūsānīs (erant autem prīncipēs mēcum) dīxī mē illud ipsum arbitrārī esse quod quaererem. Immissī cum falcibus multī pūrgārunt et aperuērunt locum:

---

conlūstrō (1): lighten on all sides; inspect
porta, -ae f.: city-gate
Agragantīnus, -a, -um: of Acragas (a Greek city in Sicily)
frequentia, -ae f.: multitude
advertō, -ere, -ī, -versus: turn to; (here, with *animum*) turn one's mind to, notice
columella, -ae f.: little column (diminutive of *columna, ae f.*)
multum (adv.): much
dūmus, -ī m.: bramble
ēmineō, -ēre, -uī, –: stand out
īnsum, -esse, -fuī, –: be in
figūra, -ae f.: figure
statim (adv.): at once
prīnceps, prīncipis: first; (as a substantive) first man, chief, leader
arbitror, -ārī, -ātus sum: (here) believe
immittō, -ere, immīsī, immissus: send to, dispatch
falx, falcis f.: sickle
pūrgō (1): purge, make clean
aperiō, -īre, -uī, -tus: uncover

---

cum omnia conlūstrārem oculīs: another circumstantial *cum* clause with an imperfect subjunctive verb

illud ipsum . . . esse quod quaererem: *that (grave) was the very one which I was searching for*; clauses that are subordinate to indirect statements need a subjunctive verb (the sequence of tenses requires *quaererem* to be an imperfect verb, because *dīxī* is in the past tense)

pūrgārunt: syncopation of *pūrgāvērunt*

66. quō cum patefactus esset aditus, ad adversam basim accessimus. Appārēbat epigramma exēsīs posteriōribus partibus versiculōrum dīmidiātum ferē. Ita nōbilissima Graeciae cīvitās, quondam vērō etiam doctissima, suī cīvis ūnīus acūtissimī monumentum ignōrāsset, nisi ab homine Arpīnāte didicisset.

---

quō: to which place, whither
patefaciō, -ere, -fēcī, -factus: make open, expose
aditus, -ūs m.: entrance
adversus, -a, -um: opposite
basis, basis f.: foundation, pedestal (from the Greek βάσις)
accēdō, -ere, -cessī, -cessus: approach
appāreō, -ēre, -uī, -itus: appear
epigramma, -atis n.: inscription (from the Greek ἐπίγραμμα)
exedō, -ere, -ēdī, -ēsus: (here) corrode
posterior, posterius: latter (of two)
versiculus, -ī m.: little verse (diminutive of *versus, -ūs m.*)
dīmidiō (1): divide into two
ferē (adv.): nearly, almost
nōbilis, -e: noble
Graecia, -ae f.: Greece
cīvitās, -ātis, f.: state, city-state
quondam (adv.): once, formerly
vērō (adv.): in truth
acūtus, -a, -um: sharp; (in the sense of a "sharp" mind) intelligent
Arpīnās, -ātis: of Arpinum (the birthplace of Cicero)
discō, -ēre, didicī, –: learn, become acquainted with

---

quō cum patefactus esset aditus: circumstantial *cum* clause with a pluperfect subjunctive verb; as a connective to the previous sentence, *quō* can precede the conjunction *cum*

exēsīs posteriōribus partibus versiculōrum: an ablative of description bracketed by the noun (*epigramma*) that it is describing and a participial phrase (*dīmidiātum ferē*) that is also helping to describe *epigramma*

ignōrāsset, nisi ab homine Arpīnāte didicisset: a past contrary-to-fact conditional with pluperfect subjunctive verbs *ignōrāsset* (a syncopation of *ignōrāvisset*) and *didicisset*; note that *ab* here means "from" in the sense of learning "from" someone; *homine Arpīnāte* refers to Cicero himself



Figure 2: Painting by Benjamin West representing Cicero's rediscovery of Archimedes's tomb. The depiction of the tomb's finial reflects neither the diagram used in Archimedes's derivation of the volume and surface area of the sphere, nor, if Plutarch is to be believed, his request that, on his tomb, "the cylinder [should be] circumscribing the sphere within" (τὸν περιλαμβάνοντα τὴν σφαῖραν ἐντὸς κύλινδρον, *Vit.* 17.7). Image credits: Yale University Art Gallery, public domain.

Dē Nātūrā Deōrum Liber Tertius

VIII. [. . .] Sed cum mundō negās quicquam esse melius, quid dīcis "melius?" Sī "pulchrius," assentior; sī "aptius ad ūtilitātēs nostrās," id quoque assentior; sīn autem id dīcis, "nihil esse mundō sapientius," nūllō modō prōrsus assentior, nōn quod difficile sit mentem ab oculīs sēvocāre, sed quō magis sēvocō, eō minus id quod tū vīs possum mente comprehendere.

---

mundus, -ī m.: world, universe (cf. French "monde")
quisquam, quidquam (quicquam): anyone, anything
melior, -ius: better (comparative of *bonus, -a, -um*)
dīcō, -ere, dīxī, dictus: (here equivalent to *intellegō*, with double accusative) mean (by)
assentior, -īrī, -sēnsus sum: agree (with)
aptus, -a, -um: suitable, adapted
ūtilitās, -ātis f.: use
sīn: but if
prōrsus (adv.): (here, intensifying a negative) at all
sēvocō (1): separate
volō, velle, voluī, –: (here) mean
comprehendō, -ere, -ī, comprehēnsus: (here) understand, comprehend

---

cum mundō negās quicquam esse melius: temporal *cum* clause with a present indicative verb; *mundō* is an ablative of comparison; the speaker is the skeptic Cotta, and he addresses the Stoic Balbus

quid dīcis "melius?": *what do you mean by "better?"*; *melius*, as well as the subsequent *pulchrius* and *aptius*, are comparative adjectives

id dīcis: the pronoun refers to the following indirect statement

nōn quod: *quod* is not a relative pronoun and instead means "because"

sit: subjunctive because Cicero thinks that it is *not* the case that separating the mind (logical reasoning) and eyes (observation) is difficult; indeed, separating the two was a common practice in Ancient Greek philosophy: for instance, the 5th-century BCE philosopher Parmenides attempted to prove through logical arguments that changes could never occur, thereby showing that the senses are deceiving

quō magis . . . eō minus: an idiom for "the more . . . the less"; literally, this expression would translate as "less by (the degree) by which . . . more," where *eō* is the antecedent of *quō* and both are ablatives of degree of difference

mente: ablative of means

IX. "Nihil est mundō melius in rērum nātūrā." Nē in terrīs quidem urbe nostrā; num igitur idcircō in urbe esse ratiōnem, cōgitātiōnem, mentem putās aut, quoniam nōn sit, num idcircō exīstimās formīcam antepōnendam esse huic pulcherrimae urbī, quod in urbe sēnsus sit nūllus, in formīcā nōn modo sēnsus, sed etiam mēns, ratiō, memoria?

---

igitur: therefore
idcircō: for that reason
ratiō, -ōnis f.: reason, logic
cōgitātiō, -ōnis f.: thought
quoniam: since, seeing that
exīstimō (1): consider, think
formīca, -ae f.: ant (cf. French "fourmi")
antepōnō, -ere, -posuī, -positus (+ dat.): (lit.) set before; prefer over
sēnsus, -ūs m.: feeling, sense, faculty of perceiving
nōn modo . . . sed etiam: not only . . . but also
memoria, -ae f.: memory

---

rērum nātūrā: a common philosophical phrase for either the "universe" itself or its properties; it appears in the titles of both Cicero's own work (*Dē Nātūrā Deōrum*) and Lucretius's epic poem (*Dē Rērum Nātūrā*), excerpted below

Nē in terrīs quidem urbe nostrā: *Not even (anything better) on earth than our city*, i.e. Rome; there exists a parallelism between *mundō* and *urbe nostrā* (both ablatives of comparison) and *in rērum nātūrā* and *in terrīs*, with the reversed order exemplifying chiasmus

num: introduces a question where the expected answer is "no"

quoniam nōn sit: the subjunctive is used to express the counterfactuality of the clause

antepōnendam esse: a passive periphrastic and the infinitive of the indirect statement with subject *formīcam*

quod: "because" with the subjunctive *sit* and not a relative pronoun

Vidēre oportet, Balbe, quid tibi concēdātur, nōn tē ipsum quod velīs sūmere. Istum enim locum tōtum illa vetus Zēnōnis brevis et, ut tibi vidēbātur, acūta conclūsiō dīlātāvit. Zēnō enim ita conclūdit: "Quod ratiōne ūtitur, id melius est quam id quod ratiōne nōn ūtitur; nihil autem mundō melius; ratiōne igitur mundus ūtitur."

---

oportet, -ēre, -uit, – (+ inf.): (impers.) it is necessary to

Balbus, -ī m.: Balbus (see the author introduction)

concēdō, -ere, -cessī, -cessus (+ dat.): assent to, concede to

sūmō, -ere, -psī, -ptus: take, receive; assume; affirm

locus, -ī m.: (here) the form or outline of an argument (used by Cicero as a literal translation of the Greek philosophical term τόπος)

vetus, veteris: old (cf. veteran)

Zēnō, -ōnis m.: Zeno of Citium (Latinization of Ζήνων; see below)

conclūsiō, -ōnis f.: conclusion

dīlātō (1): spread out, extend (cf. dilate)

conclūdō, -ere, -clūsī, -clūsus: (here) conclude

ūtor, ūtī, ūsus sum (+ abl.): use

Vidēre oportet . . . quid tibi concēdātur, nōn tē ipsum quod velīs sūmere: the indirect question introduced by *quid* takes the subjunctive *concēdātur*, while *velīs* is the subjunctive verb of a clause subordinate to an indirect statement; *sūmere*, like *vidēre*, is a complementary infinitive for *oportet*

Zēnōnis: the 3rd-century BCE philosopher Zeno of Citium — not to be confused with the pre-Socratic philosopher Zeno of Elea — founded the Stoic school of philosophy and believed that a god existed and was the universe itself (in the form of the ἀρχή or fundamental element, fire). His ontological argument goes, "A man can honor a god; one cannot honor those who are non-existent; therefore, a god must exist." The kind of counter-argument that Cicero's Cotta in the excerpt and many other philosophers used in response to Zeno is known as a παραβολή, where the overall structure of an argument is preserved but minor predicates are replaced (e.g. "rational" → "is educated") to derive more questionable conclusions (e.g. that the universe is educated and even capable of everything) and thereby raise doubts about the goodness of the original argument (Schofield, 1983)

id melius est quam id: the former *id* refers to the previous relative clause, while the latter *id* refers to the subsequent relative clause; notice that the comparison uses *quam*

Hoc sī placet, iam efficiēs ut mundus optimē librum legere videātur; Zēnōnis enim vestīgiīs hōc modō ratiōnem poteris conclūdere: "Quod litterātum est, id est melius quam quod nōn est litterātum; nihil autem mundō melius; litterātus igitur est mundus." Istō modō etiam disertus et quidem mathēmaticus, mūsicus, omnī dēnique doctrīnā ērudītus, postrēmō philosophus erit mundus.

---

placeō, -ēre, -uī, -itus (+ dat.): please, be pleasing

efficiō, -ere, -fēcī, -fectus: work out

optimē (adv.): best, very well (comparative of *benē (adv.)*)

vestīgium, -iī n.: footprint, step (cf. vestige)

litterātus, -a, -um: learned (lit. "man of letters," from *littera, -ae f.*)

disertus, -a, -um: skillful in speaking, eloquent

mathēmaticus, -ī m.: mathematician (from the Greek μαθηματικός, ultimately from μανθάνειν, "to learn"; in both Latin and Greek, the word can frequently mean "astrologer")

mūsicus, -ī m.: musician, poet (from the Greek μουσικός, "of the Muses")

dēnique (adv.): finally; and then

doctrīna, -ae f.: teaching, science, area of knowledge

ērudītus, -a, -um (+ abl.): learned in, educated in

postrēmō (adv.): at last, finally

philosophus, -ī m.: philosopher (from the Greek φιλόσοφος, lit. "lover of wisdom")

Hoc sī placet: a connective to a previous clause or sentence (in this case, the demonstrative *hoc*) can sometimes precede *sī* even if it is a part of the protasis

ut mundus optimē librum legere videātur: the verb *efficere* can take a noun clause

vestīgiīs: ablative of means

disertus et quidem mathēmaticus, mūsicus, . . . ērudītus, . . . philosophus: in this instance of asyndeton, Cicero alternates between nouns and adjectives

Chapter 2 — Lucretius: *Dē Rērum Nātūrā*

Perhaps as a subtle allusion to the philosopher and poet Titus Lucretius Carus (possibly 99 BCE – 55 BCE), Virgil wrote in his Georgics, "Happy (was) he who was able to learn the causes of things" (*Fēlīx quī potuit rērum cognōscere causās*, *Georgica* 2.490). Accordingly, one must imagine the historians studying Lucretius's life very unhappy, for not even his exact birth year is known! His magnum opus and only surviving work is the *Dē Rērum Nātūrā*, an extensive six-book, 7400-line philosophical epic poem written in dactylic hexameters. It is one of the few primary sources on the natural philosophy of Epicurus, with Epicurus's own works only extant in fragments. Although Epicurus did believe that gods exist in some way, he argued that they do not care about human affairs (as espoused by the Epicurean Gaius Velleius in Cicero's *Dē Nātūrā Deōrum*), nor do they interfere with physical phenomena. Such a belief system requires purely scientific explanations of nature, as explained in lines 146-158 excerpted from Book 1 below. Unsurprisingly, the deistic perspectives of Epicurus drew much criticism from later Christian and Jewish scholars (cf. אֶפִּיקוֹרוֹס, *epikōrōs*, a Hebrew term for a heretic).

The chosen excerpt focuses on a core Epicurean principle, that "from nothing, nothing is made" (*ex nihilō nihil fit*). Ancient Greek philosophers first developed this tenet in the context of the notorious problem of change (that is, the 5th-century BCE philosopher Parmenides's argument that change cannot occur, which relied on the implausibility of the discrete jump from "being" to "non-being," or vice-versa). Lucretius's justification (see lines 159-168 below) states that, if something could come from nothing, the process would occur constantly, for instance allowing complex lifeforms like cat-

tle, lizards, or humans to appear fully-grown everywhere, a phenomenon that is never observed. This section is a typical example of an argument *ad absurdum* and follows the *modus tollēns* structure, as outlined in more detail in the commentary: if A, then B; B is false; therefore, A is false. The *ex nihilō nihil fit* principle has persisted in Newtonian physics through mathematical conservation laws, for instance Descartes's conservation of motion and momentum, Leibniz's conservation of energy, and, of course, conservation of mass. Even more fundamentally, the theorems of Emmy Noether associate conservation laws with symmetries of nature (e.g. that the laws of physics do not change through motion, displacement in time, or rotations), a concept pervading the theoretical physics of today (Trépanier, 2023).

The *ex nihilō nihil fit* principle is central to Lucretius's defense of atomism, where changes in objects occur from the motions of indivisible, unchanging, and eternal atoms (ἄτομοι = ἀ- "not" + τέμνειν "to cut") traveling through otherwise empty void (κενόν). The theory, with atoms that can collide and then connect using hooks, was first developed by the 5th-century BCE mentor-student pair Leucippus and Democritus. Note that Lucretius avoids calques of the Greek terminology, like *atomus*, and instead exclusively employs preexisting Latin vocabulary, such as in the terms *genitālia corpora* ("generative bodies"), *corpora prīma* ("first bodies"), or *sēmina* ("seeds"), which all emphasize the generative potential of his fundamental particles.

For over a millennium, Lucretius's atomism was largely eclipsed by the Aristotelian theory of matter. In his *On Creation and Destruction*, Aristotle had rejected the existence of void by stating that the motion of an object is inversely proportional to the density of

the medium surrounding it. In place of atomism, Aristotle promoted his own theories of homogeneous elements and infinitely divisible space, time, and matter. Aristotle's works ultimately formed the foundation of medieval Islamic and European science and philosophy, while atomism in its ancient form, along with much of the *Dē Rērum Nātūrā*, gradually faded from the canon of scientific thought, although surviving in societies like Henry of Harclay's indivisibilists at 14th-century Oxford (Murdoch, 2009). Eventually, at a time when many Ancient Greek philosophical works were reintroduced in the lead up to the Renaissance, the rediscovery of a manuscript of the *De Rerum Natura* by humanist Poggio Bracciolini in 1417 gave new life to the theory of atomism, in various flavors: Pierre Gassendi's God-created atoms, which were a compromise with the Christian doctrine; Leibniz's indivisible monads, which echo the principles of his combinatorial art and universal language (see the chapter on Leibniz); Descartes's and Boyle's infinitely divisible and continuous corpuscles, which provided a physical justification for Newton's calculus; and, more recently, the chemists' billiard ball atoms or Neils Bohr's fuzzy quantum atoms of probability (Jensen, 2010). Ironically, the modern notion of an atom departs from the word's aforementioned Greek etymology, since what we now call atoms are composed of even smaller particles.

Source Text: Our Latin text comes from Leonard and Smith (2008).

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Dē Rērum Nātūrā Liber Prīmus

Nam tibi dē summā caelī ratiōne deumque 54
disserere incipiam et rērum prīmōrdia pandam, 55
unde omnīs nātūra creet rēs, auctet alatque, 56
quōve eadem rūrsum nātūra perēmpta resolvat, 57

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ratiō, -ōnis f.: reason, logic; (here) law, order
disserō, -ere, -uī, -tus (+ *dē*): arrange; examine, discuss (cf. dissertation)
incipiō, -ere, -cēpī, -ceptus: begin
prīmōrdium, -iī n.: origin (cf. primordial)
pandō, -ere, -ī, passus: open, extend; explain
unde: from which place, from which, wherefrom
creō (1): bring forth, create
auctō (1): increase (cf. auction)
alō, -ere, -uī, -tus: support, maintain; promote, increase
quō: to which place, whither (in contrast to *unde*)
-ve: or
rūrsum (adv.): again
perimō, -ere, -ēmī, -ēmptus: annihilate, destroy; prevent
resolvō, -ere, -ī, -solūtus: untie, release; dissolve, destroy

---

tibi: refers to Gaius Memmius, to whom Lucretius dedicated his poem

deumque: *deum* is the genitive plural of *deus* and an alternative form of *deōrum*

omnīs: *-īs* is a variant of the usual accusative plural ending *-ēs* that appears frequently in poetry; *omnīs* modifies *rēs*, which is the object of *creet*, *auctet*, and *alat*

creet: when a subordinate clause is contained in an indirect statement, its verb is subjunctive

eadem: modifies *nātūra*

perēmpta: PPP modifying an implicit neuter plural *ea*

quae nōs māteriem et genitālia corpora rēbus 58
reddundā in ratiōne vocāre et sēmina rērum 59
appellāre suēmus et haec eadem ūsūrpāre 60
corpora prīma, quod ex illīs sunt omnia prīmīs. 61

---

māteriēs (acc. māteriem) f.: matter
genitālis, -e: belonging to or causing birth, generative; (here, with *corpus*) elements
reddō, -ere, -didī, ditus: return; (here with direct object *ratiō*) render an account
sēmen, sēminis n.: seed
appellō (1): name
suēscō, -ere, suēvī, suētus: be accustomed to
ūsūrpō (1): use; seize; (here) name

---

quae: technically a relative pronoun with antecedent *omnīs . . . rēs*, but can be treated as a demonstrative pronoun ("these things")

reddundā in ratiōne: *in rendering an account*; idiomatic use of the gerundive for a gerund construction

suēmus: syncopation of *suēvimus*; the verb has the complementary infinitives *vocāre*, *appellāre*, and *ūsūrpāre*, which all take a double accusative (specifically, one of the accusatives, *quae*, refers to the things being named, and the others represent the names being assigned to them)

Nec mē animī fallit Grāiōrum obscūra reperta 136
difficile inlūstrāre Latīnīs versibus esse, 137
multa novīs verbīs praesertim cum sit agendum 138
propter egestātem linguae et rērum novitātem; 139

---

fallō, -ere, fefellī, falsus: deceive; escape notice
Grāiī, -ōrum m.: the Greeks
obscūrus, -a, -um: obscure (both in the sense of "dark" and "unintelligible")
reperta, -ōrum n.: inventions (from *reperiō, -īre, -perī, -ertus*, "to find")
inlūstrō (1): light up; to make clear (cf. illustrate)
Latīnus, -a, -um: Latin
versus, -ūs m.: verse
praesertim: especially, particularly
agō, -ere, ēgī, āctus: (here) convey
egestās, -ātis f.: extreme poverty, want
novitās, -ātis f.: novelty

Nec mē animī fallit: (idiomatically) *it does not escape my mind that*; here, *animī* is not a genitive, but a locative ("in the mind")

esse: the subject of the indirect statement is the infinitive phrase *Grāiōrum obscūra reperta . . . inlūstrāre Latīnīs versibus*; as a result, the predicate adjective *difficile* has to take on the neuter gender

praesertim cum: in prose, this phrase would be located at the beginning of the clause, behind *multa novīs verbīs*

sit agendum: a passive periphrastic and the subjunctive verb of a causal *cum* clause; strictly speaking, *sit agendum* is impersonal ("it must be conveyed"), but, when translating, the accusative object *multa* of the transitive verb *agere* can be understood as the subject of *sit agendum*

egestātem linguae et rērum novitātem: *linguae* refers to the Latin language; in this line, *rērum* does not mean "things," but "subject matters," specifically the main components of the Epicurean philosophy

Hunc igitur terrōrem animī tenebrāsque necessest 146
nōn radiī sōlis neque lūcida tēla diēī 147
discutiant, sed nātūrae speciēs ratiōque. 148
Prīncipium cuius hinc nōbīs exōrdia sūmet, 149
nūllam rem ē nihilō gignī dīvīnitus umquam. 150

---

igitur: therefore
terror, -ōris m.: fear
tenebrae, -ārum f.: darkness, shadows
necessest = necesse est: it is necessary that, it is inevitable that
radius, -iī m.: (here) ray
lūcidus, -a, -um: shining
tēlum, -ī n.: weapon; (here figuratively) sunbeam
discutiō, -ere, -cussī, -cussus: strike, shatter, destroy
speciēs, -ēī f.: aspect, appearance
prīncipium, -iī n.: beginning; principle
hinc: from this place; (here) hence
exōrdium, -iī n.: beginning, introduction, exordium
sūmō, -ere, -psī, -ptus: take, receive; assume; (here) affirm
nihilum, -ī n. = nihil
gignō, -ere, genuī, genitus: bring forth, birth, produce
dīvīnitus (adv.): by divine interference
umquam: ever

---

necessest: this word sets up the subjunctive verb *discutiant*

discutiant: its four subjects are *radiī*, *tēla*, *speciēs*, and *ratiō*

Prīncipium . . . exōrdia sūmet: *Prīncipium* and *exōrdia* are very similar in their definitions (both words can mean "beginning"), but here the subject *exōrdia* (a poetic plural) describes the beginning of the text while the object *prīncipium* describes the beginning or foundation for the laws of nature

cuius: the relative pronoun *cuius* has *speciēs ratiōque* as its antecedent and is an objective genitive governed by *prīncipium*; even though *cuius* is usually written with only one *i*, its pronunciation actually features a vocalic *i* in the first syllable (*cui-*) and a consonantal *i* in the second (*-ius*); this analysis implies that the first syllable scans as long, which Lucretius, in an archaism, treats as short

gignī: it is the passive infinitive of an indirect statement that describes the *prīncipium*

Quippe ita formīdō mortālīs continet omnīs, 151
quod multa in terrīs fierī caelōque tuentur, 152
quōrum operum causās nūllā ratiōne vidēre 153
possunt ac fierī dīvīnō nūmine rentur. 154

---

quippe: surely, indeed (as an adv.); for indeed, because (as a conjunction)
formīdō, -inis f.: fear (cf. formidable)
mortālēs, -ium m.: mortals, humanity
contineō, -ēre, -uī, -tentus: keep together, enclose, subdue
tueor, -ērī, -itus sum: look at, behold, examine
opus, operis n.: work
videō, -ēre, vīdī, vīsus: (here) understand
nūmen, nūminis n.: (divine) will, power
reor, rērī, ratus sum: believe

---

mortālīs . . . omnīs: a poetic version of the accusative plural ending *-ēs*
in terrīs . . . caelōque: *on the lands and in the sky*
nūllā ratiōne: *with no logic*; ablative of means

Quās ob rēs ubi vīderimus nihil posse creārī 155
dē nihilō, tum quod sequimur iam rēctius inde 156
perspiciēmus, et unde queat rēs quaeque creārī 157
et quō quaeque modō fiant operā sine dīvom. 158

---

quās ob rēs: (lit.) because of these things, therefore
rēctus, -a, um: straight, right, proper
inde: from that place, then
perspiciō, -ere, -spexī, -spectus: look through; perceive, ascertain
queō, -īre, -īvī, -itus: be able to
quō . . . modō: in what way
dīvus, -a, -um: divine; (as a substantive) god

---

quod: a relative pronoun that, in prose, would follow the *iam*

rēctius: *more properly*; comparative adverb from *rēctus, -a, um*

et unde queat rēs quaeque creārī / et quō quaeque modō fiant: *queat* and *fiant* are subjunctive because they are verbs in indirect questions introduced by *unde* and *quō . . . modō*, respectively; Lucretius is thereby introducing the two major questions he seeks answer in his work with the help of atoms, whose existence he will establish after having proven that nothing can come from nothing: how are composite bodies created from fundamental elements, and how can phenomena be explained purely scientifically, without religion?

quaeque: in the first line, *quaeque* is feminine nominative singular and modifies *rēs*, in contrast to the neuter plural nominative *quaeque* in the second line

dīvom: an archaic genitive plural for *dīvus*

Nam sī dē nihilō fierent, ex omnibus rēbus 159
omne genus nāscī posset, nihil sēmine egēret. 160
Ē mare prīmum hominēs, ē terrā posset orīrī 161
squāmigerum genus et volucrēs ērumpere caelō; 162
armenta atque aliae pecudēs, genus omne ferārum, 163
incertō partū culta ac dēserta tenērent. 164

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genus, generis n.: (here) offspring
nāscor, -ī, nātus sum: be born, be produced
egeō, -ēre, -uī, – (+ abl.): need
prīmum (adv.): first
orior, -īrī, -tus sum: rise, appear
squāmiger, -era, -erum: scaly
volucris, -is f.: bird
ērumpō, -ere, -rūpī, -ruptus (+ abl.): burst forth from, break out from
armentum, -ī n.: cattle
pecus, pecudis f.: animal, farm animal (in contrast to *fera*)
fera, -ae f.: wild beast (cf. *ferōx, -ōcis*)
incertus, -a, -um: uncertain
partus, -ūs m.: offspring; birth
colō, -ere, -uī, -tus: cultivate
teneō, -ēre, -uī, -tus: (here) occupy, possess, fill

---

fierent . . . posset . . . egēret: the imperfect subjunctives in the protasis and apodosis indicate a contrary-to-fact present conditional; translate the verb in the protasis with "were" and the verbs in the apodosis with "would"; note that *posse* in the subjunctive can often be translated simply as "could"; this conditional, which continues onto the next page with the imperfect subjunctives *solērent*, *mūtārentur*, and *possent*, forms the "if A, then B" premise of the *modus tollēns* argument

ē mare prīmum hominēs: understand *possent orīrī*; *mare* is a somewhat rare alternative to *marī* for the ablative singular

Nec frūctūs īdem arboribus cōnstāre solērent, 165
sed mūtārentur; ferre omnēs omnia possent. 166
Quippe ubi nōn essent genitālia corpora cuique, 167
quī posset māter rēbus cōnsistere certa? 168

---

frūctus, -ūs m.: fruit
arbor, -oris f.: tree
cōnstō, -āre, -stitī, -stātūrus (+ dat.): correspond to
mūtō (1): change
quī: (as an interrogative adv.) in what manner, how
cōnsistō, -ere, -stitī, –: be constant, remain unchanged
certus, -a, um: fixed, unerring, constant

---

omnēs: modifies an implicit *arborēs*

ubi nōn essent genitālia corpora cuique: a dative of possession that can be rephrased as *ubi quidque nōn habēret genitālia corpora*; *ubi* sometimes takes a subjunctive when used temporally as "when," and this line and the next can be treated as a contrary-to-fact present conditional like above

quī posset māter rēbus cōnsistere certa: *how could a mother for things remain fixed*; the redundancy of *cōnsistere certa* is an instance of the literary device of pleonasm

At nunc sēminibus quia certīs quaeque creantur, 169
inde ēnāscitur atque ōrās in lūminis exit, 170
māteriēs ubi inest cuiusque et corpora prīma; 171
atque hāc rē nequeunt ex omnibus omnia gignī, 172
quod certīs in rēbus inest sēcrēta facultās. 173

ēnāscor = nāscor
ōra, -ae f.: extremity, region
īnsum, -esse, -fuī, –: be in, be contained in
nequeō, -īre, -īvī, -itus: cannot
sēcrētus, -a, -um: separate, distinct
facultās, -ātis f.: power, ability, faculty

At nunc: with these words, Lucretius announces that he is transitioning into the "B is false" premise of the *modus tollēns* argument

quia: in prose, it would be placed before *sēminibus*

ōrās in lūminis: just as a preposition can sometimes be placed in between a noun and an adjective (e.g. *magnā cum laude*), it can also lie between a noun and a genitive governed by the noun; *orae lūminis* metaphorically means "the world," which indeed is the "region" of "light" and life, as opposed to the underworld

māteriēs ubi inest cuiusque et corpora prīma: *in which the matter and the primary bodies of anything are contained*; although *inest* also has a plural subject *corpora prīma*, it is singular because its first subject, *māteriēs*, is singular (which is common if a verb has multiple subjects, some singular, some plural)

hāc rē *therefore* (literally "because of this thing," an ablative of cause); Lucretius finally reaches the "A is false" conclusion of his argument, although it is slightly stronger than the statement that he initially set out to prove: Rather than simply demonstrating that nothing can come from nothing, he also deduces that a certain *facultās* — namely, atoms and their motions — prevents something from emerging from any random source or at any random time

Chapter 3 — Copernicus: *Commentāriolus*

Nicolaus Copernicus (1473 CE – 1543 CE) is known for introducing the heliocentric planetary model and inspiring scientists like Galileo Galilei and Isaac Newton to develop new theories of motion through his monumental *Dē Revolūtiōnibus Orbium Coelestium*. At the urging and with the assistance of his pupil Georg Joachim Rheticus, who ultimately became one of the editors of the work, Copernicus had it published but, contrary to popular belief, did not intentionally postpone the publication until after his death to avoid persecution. Instead, it is said that he had the opportunity to look at the printed version on the day of his passing (Rabin, 2023). In a modern version of Cicero's search for Archimedes's tomb, archaeologists only recently rediscovered the astronomer's burial place in the Frombork Cathedral, having unearthed his skull and compared its DNA to that of hair contained in a book once owned by him (Easton, 2008)!

The ideas that Copernicus would eventually compile into *Dē Revolūtiōnibus* in the 1530s were already germinating, and receiving recognition, decades earlier. The excerpt below serves as the beginning to a manuscript that Tycho Brahe later titled the *Commentāriolus* (Rosen, 1992),[1] literally the "little commentary," in what is perhaps the biggest understatement in the history of astronomy, not because of its length (a compact 16 pages), but its impact. After Copernicus outlines the problems present in earlier geocentric systems, he proposes a new set of premises that lead to a simpler description of planetary motion. As he himself admits, the rigorous mathematical constructions and computations are "intended for a greater volume" (*maiōrī volūminī dēstinātās*), but the text does provide some insight as to why he might have pursued

[1] Full Latin title: *Dē Hypothesibus mōtuum coelestium ā sē cōnstitūtīs Commentāriolus*

a heliocentric theory in the first place. Even if Copernicus only shared the work with a few close astronomers sometime before 1514, it did attract the attention of many important figures, not only the aforementioned Tycho, but also Pope Clement VII and Cardinal von Schönberg, who, perhaps unexpectedly, encouraged him to publish a full-length work on the subject (Macke, 2020). The original draft of the *Commentāriolus* is unfortunately lost, but the historian of mathematics Maximilian Curtze discovered the first of three currently known copies in the Austrian National Library in 1878 (Rosen, 1992).

As Copernicus's frequent references to previous astronomers in the *Commentāriolus* suggest (each highlighted in the commentary below), his work closely engages with earlier Greek astronomical theories. Most importantly, it upended the Aristotelian worldview by assuming that the Earth does not remain fixed at the center of the universe. While Aristotle framed geocentrism as a corollary of the core principle that earth and water tend to move towards the center of the universe, later philosophers also devised other arguments in defense of Earth's immobility. For instance, since objects thrown into the air from a moving vehicle would intuitively land behind it, objects thrown upwards from the surface of the Earth would never land at the same place if the Earth were moving (Ptolemaeus, 1984). (It turns out that this absurd implication of a moving Earth is resolved by the conservation of momentum devised in post-Renaissance physics.) Copernicus's rejection of geocentrism implied that most Western scientific knowledge, largely derived from Aristotle, stood on, literally, moving ground, which is precisely what makes this paradigm shift, since referred to as the "Copernican revolution," deserving of that title.

As profound as its philosophical implications may have been, Copernicus's system actually reused much of the machinery of the older geocentric Ptolemaic model, including letting planets move on circles (epicycles) whose centers themselves move on larger circles (deferents). Despite this mathematical complexity, Copernicus did formulate two interesting aesthetic justifications for his heliocentric model: The first, touched upon in this excerpt of the *Commentāriolus*, is the removal of the so-called equant point, a mathematical invention that was not believed to exist in real life but rather was invented to preserve a core belief of the Aristotelian worldview, that planets move at a constant speed (Rabin, 2023). The second, expounded in the first book of the *Dē Revolūtiōnibus*, is the implementation of a relationship that had been envisioned earlier by Aristotle and even the Roman architect Vitruvius, namely that a planet's period corresponds to the radius of its orbit. In contrast, in the Ptolemaic model, the order of the planets, especially the innermost ones, was arbitrary (Goldstein, 2002).

Just as his preferred defense for heliocentrism changed over time, his astronomical results were revised and reframed by the editors of the *Dē Revolūtiōnibus* as well, not always with his approval. Rheticus worked alongside Copernicus in his final years and expanded upon the mathematical aspects of the theory to align them with contemporary texts (in fact, Rheticus published a whole separate treatise meant to assist in the trigonometric calculations of the Copernican system). He also authored the 1540 *Nārrātiō prīma*, which, as its name suggests, was the first public, printed document concerning Copernicus's heliocentrism. It warned the reader that some of the necessary assumptions (namely the mobility of the Earth) might contradict philosophical reasoning. It was eventually incorporated

into the second edition of the *Dē Revolūtiōnibus* and even republished in Johannes Kepler's *Mystērium Cosmographicum* (Rabin, 2023). However, the preface that would appear in the first edition of the *Dē Revolūtiōnibus* was not only cautious, but also downplayed the potentially controversial elements of the heliocentric system — including the main point of the text, that the Earth is in motion — as mere hypotheses that are considered true for the sake of astronomical calculations but do not accurately reflect reality. Its author was the minister Andreas Osiander, who took over as editor after Rheticus received a teaching position at the University of Leipzig. Rheticus himself was very offended at this introduction, striking it out in red ink in his own copy of the *Dē Revolūtiōnibus*, and this recontextualization of the Copernican model by Osiander is now known in the philosophy of science as an early example of an instrumentalist theory that is supposed to match only observations and not the nature of the universe itself.[2] On the bright side, Osiander's conservatism perhaps helped Copernicus's ideas spread throughout 16th-century Europe and his model be temporarily accepted as a valid mathemat-

[2] The following paragraph from Copernicus's introduction to the *Dē Revolūtiōnibus* sheds some light on the intensity of his own position, although it must, of course, be read with the understanding that he dedicated his work to the Pope: *Sī fortasse erunt* ματαιόλογοι*, quī cum omnium Mathēmatum ignārī sint, tamen dē illīs iūdicium sibi sūmunt, propter aliquem locum scrīptūrae, male ad suum prōpositum dētortum, ausī fuerint meum hoc īnstitūtum reprehendere ac īnsectārī: illōs nihil moror, adeō ut etiam illōrum iūdicium tanquam temerārium contemnam. Nōn enim obscūrum est Lactantium, celebrem aliōquī scrīptōrem, sed Mathēmaticum parum, admodum puerīliter dē fōrmā terrae loquī, cum dērīdet eōs, quī terram globī fōrmam habēre prōdidērunt. Itaque nōn dēbet mīrum vidērī studiōsīs, sī quī tālēs nōs etiam rīdēbunt. Mathēmata mathēmaticīs scrībuntur . . .* ("If there are possibly idle-talkers, who, although they are ignorant of all Astronomy, nevertheless assume for themselves a judgement about those (matters), they will dare to reprehend and censure this undertaking of mine because of some passage of scripture, badly distorted to their purpose: I care nothing about those men, to the extent that I disregard even the judgement of those men as reckless. It is also not unknown that Lactantius, otherwise a celebrated writer, but hardly an Astronomer, speaks very childishly about the shape of the Earth, when he derides those who made known that the Earth has the shape of a globe. Therefore, it must not seem surprising to scholars, if some people of this sort also laugh at us. Astronomy is written for astronomers . . ."). Note that Lactantius was a 4th-century Christian writer whose style was greatly admired during the Renaissance.

ical alternative to the Ptolemaic one, at least until the 1632 Galilean affair that led to the Catholic Church prohibiting the reading of the *Dē Revolūtiōnibus* and other writings on heliocentrism (Gingerich, 1973). Gradual investigations into the physical causes behind the planetary motions eventually supplied a theoretical basis for Copernicanism, with the Church repealing its ban in the early 19th century, but that story is best told in the context of Kepler and Newton's excerpts later in the reader.

Source Texts: Our Latin text comes from Folkerts et al. (2019), with a few divergences, some restoring readings present in the three original manuscripts reprinted in Copernicus (1992):

Page 77	*ex*	*et*
Page 78	*Calippus*	*Calipus*
Page 78	*et*	*ex*
Page 82	*Ptolemaeō*	*Ptolemēō*
Page 87	*paene*	*pēne*
Page 91	*circā Sōlem volvimur*	*circumvolvimur*

Note: In Latin, many verbs of motion are exclusively transitive and always require an object. So, to describe an action in which the subject is both doing the action and is affected by it, such verbs would have to take a reflexive pronoun *sē*. However, in the context of pre-Newtonian astronomy, the planets are not actually responsible for their own movement across space. Therefore, in the excerpt below, verbs of motion are frequently in the passive, although an English translation would treat them as active and intransitive.

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Dē Hypothesibus mōtuum coelestium ā sē cōnstitūtīs Commentāriolus

Multitūdinem orbium coelestium maiōrēs nostrōs eam maximē ob causam posuisse videō, ut appārentem in sīderibus mōtum sub rēgulāritāte salvārent. Valdē enim absurdum vidēbātur coeleste corpus in absolūtissimā rotunditāte nōn semper aequāliter movērī.

---

hypothesis, -is: hypothesis (from the Greek ὑπόθεσις, something "set before" or "suggested")

mōtus, -ūs m.: movement, motion

coelestis = caelestis, -e: celestial

cōnstituō, -ere, -ī, -ūtus: set up, establish

commentāriolus, -ī m.: little commentary (diminutive of *commentārius, -ī m.*)

multitūdō, -inis f.: great number, multitude

orbis, -is m.: orb, sphere, circle

maiōrēs, maiōrum m.: ancestors

maximē (adv.): (here) mostly, especially

ob (prep. + acc.): on account of, because of

pōnō, -ere, posuī, positus: (here) establish, propose

appārēns, -entis: apparent

sīdus, sīderis n.: star

sub (prep. + abl.): (here) by means of, through

rēgulāritās, -ātis f.: regularity

salvō (1): save (in order to "save the phenomena," σῴζειν τὰ φαινόμενα, astronomers should develop theories that are consistent not only with the observed planetary motions but also with the philosophical principles of the Aristotelian worldview, e.g. that orbits are perfect and circular)

valdē (adv.): very

absurdus, -a, -um: absurd

absolūtus, -a, -um: complete; perfect (from *absolvō, -ere, -ī, -solūtus*; here in the superlative)

rotunditās, -ātis f.: roundness; round figure

aequāliter (adv.): equally, uniformly

moveō, -ēre, mōvī, mōtus: move

Multitūdinem . . . posuisse: indirect statement with subject *maiōrīes nostrōs*, perfect active infinitive *posuisse*, and object *multitūdinem*

eam maximē ob causam: *eam* functions as a demonstrative adjective modifying *causam*, and, like other demonstratives, can refer to a subsequent clause often introduced by *ut* (here with the subjunctive verb *salvārent*, which is imperfect because *posuisse* is in the past tense)

Valdē enim absurdum vidēbātur: *For it seemed very absurd that*; introduces an indirect statement with subject *coeleste corpus* and infinitive *movērī*

movērī: *move*; a passive verb of motion with an active, intransitive meaning

Fierī autem posse animadverterant, ut et compositiōne atque concursū mōtuum rēgulārium dīversimodē ad aliquem situm movērī quippiam vidērētur.

---

animadvertō, -ere, -ī, -versus: (lit.) direct one's mind to; notice, observe
compositiō, -ōnis f.: arrangement, composition, combination
concursus, -ūs m.: (here) combination
rēgulāris, -e: regular
dīversimodē (adv.): in different ways
situs, -ūs m.: position, place
quispiam, quaepiam, quidpiam (quippiam): anyone, anything

Fierī . . . ut: an idiom for "it happens that"; to be precise, *ut* introduces a noun clause that functions as the subject of *fierī*

animadverterant: the subject is the "our ancestors" from earlier; because this verb is in the past tense, the subjunctive *vidērētur* in the noun clause introduced by *ut* must be imperfect; Copernicus seems to be referencing Plato in particular, who was first to suggest that the irregular planetary motions may arise from the combinations of regular, circular motions ("saving the phenomena")

et compositiōne atque concursū: ablatives of means

Id quidem Calipus et Eudoxus per concentricōs circulōs dēdūcere labōrantēs nōn potuērunt ex hīs omnium in mōtū sīdereō reddere ratiōnem, ⇒

Calipus, -ī m.: Callippus (slightly corrupt Latinization of Κάλλιππος; see below)

Eudoxus, -ī m.: Eudoxus of Cnidus (Latinization of Εὔδοξος; see below)

concentricus, -a, -um: concentric

circulus, -ī m.: circle; (in astronomy) a circular course

dēdūcō, -ere, -dūxī, -ductus: (here) derive

labōrō (1): work, strive (cf. labor)

sīdereus, -a, -um: pertaining to the stars, sidereal

reddō, -ere, -didī, ditus: return; (here with direct object *ratiō*) render an account

---

id: object of *dēdūcere* and refers to the goal of describing the planetary motions through circular motions

Calipus et Eudoxus: Eudoxus of Cnidus was an Ancient Greek mathematician and astronomer from the 4th century BCE famous for creating one of the first reasonably accurate models of planetary motion; it was composed of 3-4 concentric spheres for each body, with the poles of each sphere attached to an outer sphere, the Earth placed at their center, and the planet affixed somewhere on the innermost sphere (see Figure 1); the first version of the model was incapable of describing the paths of Venus and Mars, but the astronomer Callippus, as Eudoxus's mentee, added 7 spheres to the prior 27 to improve the model's accuracy (Aristotle then later almost doubled the number of spheres to 56!)

labōrantēs: a plural PAP modifying both subjects *Calipus* and *Eudoxus*; it has a concessive force and governs the supplemental infinitive *dēdūcere*

nōn potuērunt ex hīs omnium in mōtū sīdereō reddere ratiōnem: *were not able to render an account for all things in* (or "aspects of") *the motion pertaining to the stars by means of these (circular courses)*; *omnium* is an objective genitive with *ratiōnem* and *hīs* an ablative of means; the preposition *ex* is being used instrumentally here, but a couple of manuscripts instead read *et*, which should still be taken closely with *hīs* with the sense of "even"

in mōtū sīdereō: *sīdus* and its derivatives can refer not only to true stars but also to planets

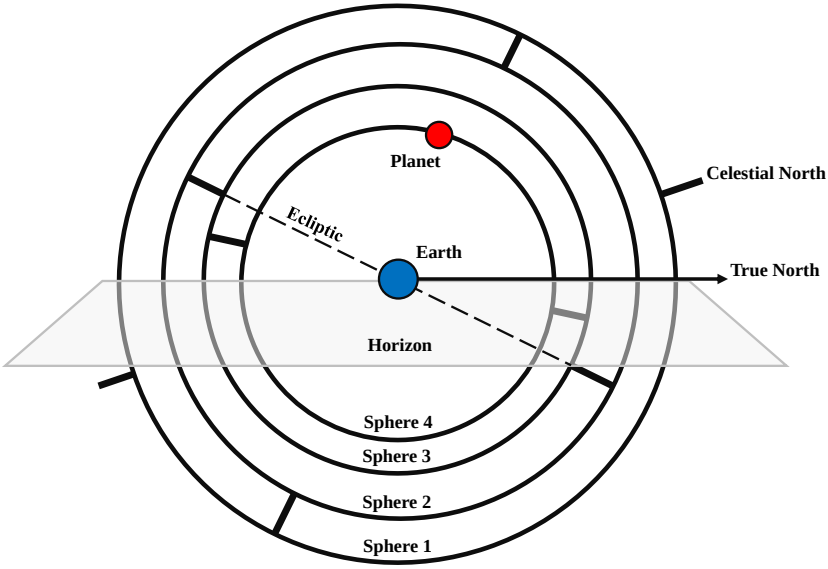


Figure 1: The Eudoxian geocentric model for a single planet. Sphere 1 has its axis directed towards the celestial north (i.e. Polaris) and its rotation describes the motions of the stars caused by the rotation of the Earth. Sphere 2 has its axis attached to Sphere 1 and its equator defines the ecliptic, along which the Sun travels. The axis of each subsequent sphere is attached to the previous sphere. The planet is then located on Sphere 4 (or Sphere 3 in the cases of the Sun and the Moon), and its motion is the combination of all rotations.

(continued) nōn sōlum eōrum, quae circā revolūtiōnēs sīderum videntur, vērum etiam quod sīdera modo scandere in sublīme, modo dēscendere nōbīs videntur, quod concentricitās minimē sustinet. Itaque potior sententia vīsa est per eccentricōs et epicyclōs id agī, in quā dēmum maxima pars sapientium convenit.

---

nōn sōlum . . . vērum etiam: not only . . . but also
circā (prep. + acc.): near; concerning
revolūtiō, -ōnis f.: revolution, circular movement
modo . . . modo: now . . . now, at one moment . . . at another
scandō, -ere, -ī, scānsus: rise, ascend
sublīmis, -e: high, lofty; (as a neuter substantive) upper regions, heavens
dēscendō, -ere, -ī, -scēnsus: descend
concentricitās, -ātis f.: concentricity
minimē (adv.): not at all
sustineō, -ēre, -uī, sustentus: support
potior, potius: better, preferable (comparative of *potis, -e*, "able")
sententia, -ae f.: way of thinking, judgment, opinion; idea
eccentricus, -a, -um: eccentric (as a masculine substantive, refers to the larger, non-centered circle in the Ptolemaic model, known as the "deferent"; see Figure 2)
epicyclus, -ī m.: epicycle (the smaller circle attached to the deferent)
dēmum (adv.): at last; indeed
sapiēns, -entis: wise; (as a substantive) wise man
conveniō, -īre, -vēnī, -ventus: agree with, (here with *in*) agree upon

---

eōrum, quae . . . videntur: *for those things which are seen*, i.e. "those appearances"; like *omnium*, *eōrum* is an objective genitive governed by *ratiōnem* and refers back to *omnium in mōtū sīdereō*

quod . . . quod: the first *quod* means "because," while the second is a neuter accusative relative pronoun whose antecedent is the entire previous clause

modo scandere . . . modo dēscendere: by "ascend" and "descend," Copernicus does not refer to the "rising" or "setting" of the planets in the night sky, but to their varying distance from the Earth, as indicated by the fact that their perceived brightness and size change over time; while Copernicus's first issue with the Eudoxean model (its inaccuracy in describing the angular motions of the planets in the night sky) can be addressed through fine-tuning or the addition of more spheres, this second issue definitively rejects the validity of a model composed of concentric spheres, as it would always keep the planets at a fixed distance from the Earth

id agī, in quā: *id* again refers to the mission of "saving the phenomena"; *agī* is a passive infinitive in an indirect statement set up by *sententia*; the antecedent of *quā* is *sententia*

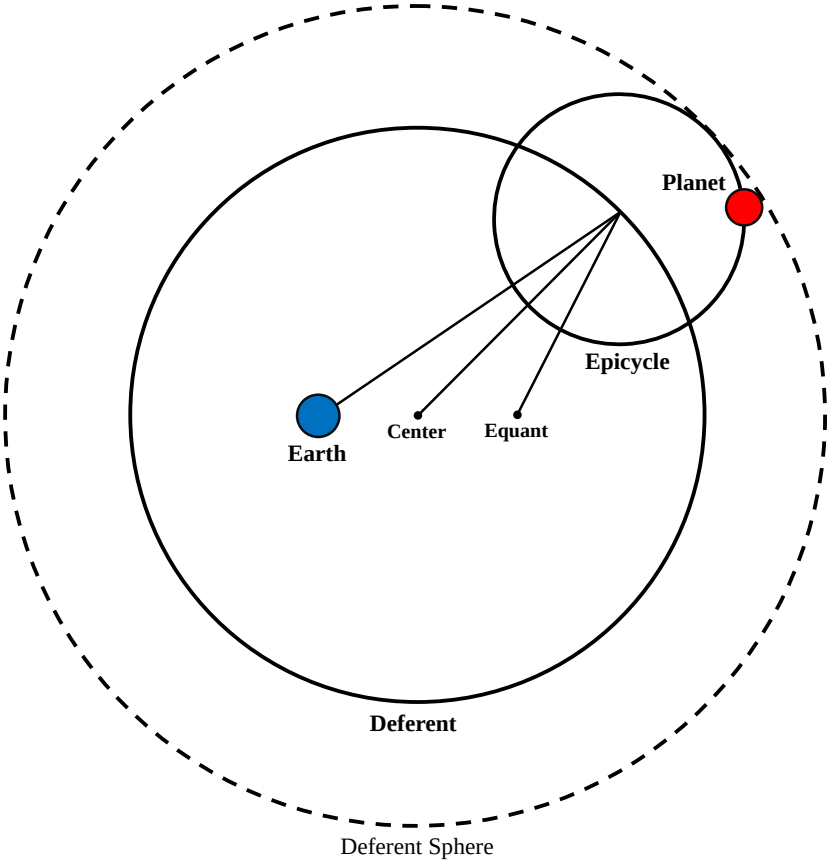


Figure 2: The Ptolemaic model for a single planet, with the center of the epicycle orbiting along the deferent, albeit not at a uniform speed. This issue is addressed by the equant, as will be discussed later in the commentary. Note that ancient astronomers still understood the circular deferent and epicycles as the equators of celestial spheres or "orbs," hence the word "orbit." (An orbit as the path of a body through space is a modern notion due to Kepler.) To avoid an unphysical intersection of spheres, the deferent was often replaced with a larger sphere with the same center, internally tangent to the epicycle.

Attamen quae ā Ptolemēō et plērīsque aliīs passim dē hīs prōdita fuērunt, quamquam ad numerum respondērent, nōn parvam quoque vidēbantur habēre dubitātiōnem.

---

attamen: but nevertheless (from *at* + *tamen*)

Ptolemēus: Claudius Ptolemy (Latinization of Πτολεμαῖος)

plērusque, -aque, -umque: most, very many

passim (adv.): here and there, far and wide, everywhere

prōdō, -ere, -didī, -ditus: put forth; make known; pass down

quamquam: although

numerus, -ī m.: number (here, the numerical data concerning the positions of the planets)

respondeō, -ēre, -ī, -spōnsus: (here, with *ad*) agree with, correspond to

dubitātiō, -ōnis f.: doubt, uncertainty

---

quae: *the things which*; the antecedent is an implicit *ea*, which functions as the subject of *vidēbantur*

Ptolemēō et plērīsque aliīs passim: the 2nd-century CE mathematician Claudius Ptolemy composed the Almagest (from the Arabic article "al" and the Greek μεγίστη "greatest," the original title being Μαθηματικὴ Σύνταξις or "A Mathematical Treatise"), which proposed the geocentric Ptolemaic model, where the planets orbit on epicycles, whose centers in turn orbit around deferent orbits; the "very many others everywhere" is ambiguous but may refer to astronomers who proposed similar multi-circle models, including perhaps the Persian astronomer Nasir al-Din al-Tusi with his Tusi couple (a supplementary system with a circle rolling around in another circle), which Copernicus seemingly incorporated into his heliocentric system (see Figures 3 & 4)

prōdita fuērunt: a common alternative to the perfect passive *prōdita sunt*

nōn parvam . . . habēre dubitātiōnem: notice the litotes in "not little uncertainty"; *habēre dubitātiōnem* can be translated literally as "to have doubt" or more smoothly as "to be doubtful" or "to require consideration"

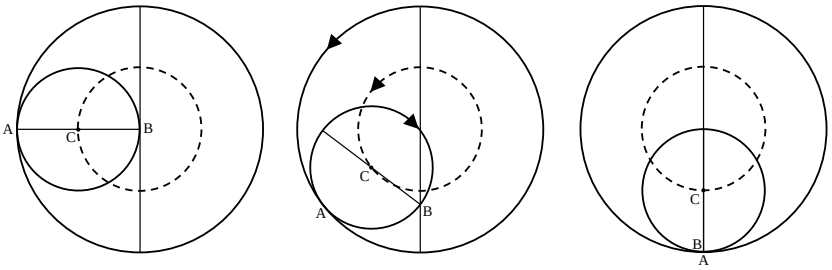


Figure 3: In a Tusi couple, an outer circle remains tangent at A to an inner circle with half the radius while they both rotate at the same angular speed and in opposite directions. One can also think of the smaller circle rolling around inside the larger one. Any fixed point B on the inner circle follows a diameter of the outer circle. Three time steps are shown. Copernicus reformulated the Tusi couple in terms of epicycles of equal size, where the center of one epicycle is affixed at C to the other epicycle (the dashed circle).

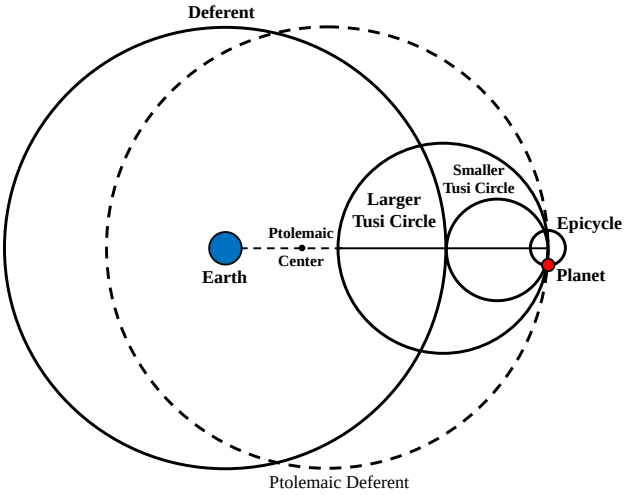


Figure 4: The integration of the Tusi couple into a geocentric model. The oscillatory motion of the center of the epicycle along a line that is directed towards the Earth produces a displaced circle (here dashed) that rotates at a non-uniform speed, while the actual celestial circles satisfy the principle of uniform motion. When one simulates this system, one can see that the center of the epicycle only follows an approximately circular path.

Nōn enim sufficiēbant, nisi etiam aequantēs quōsdam circulōs imāginārentur, quibus appārēbat neque in orbe suō dēferente neque in centrō propriō aequālī semper vēlōcitāte sīdus movērī.

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sufficiō, -ere, -fēcī, -fectus: (here) be sufficient, be adequate
aequō (1): equalize
imāginor, -ārī, -ātus sum: imagine, devise, contrive
appāreō, -ēre, -uī, -itus: appear, seem to be
dēferēns, -entis: deferent (describing the main eccentric circle of the Ptolemaic model, which is "carrying" the center of the epicycle around)
centrum, -ī n.: center (from the Greek κέντρον)
proprius, -a, -um: one's own, proper
aequālis, -e: equal; uniform
vēlōcitās, -ātis f.: velocity

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aequantēs quōsdam circulōs: in the Ptolemaic model, the equant or "equalizing point" lies opposite the Earth and at the same distance from the deferent's center; although the center of the epicycle cannot travel at a uniform speed around the deferent, which contradicts a core belief of the Aristotelian worldview, it does travel at uniform (angular) speed from the perspective of the equant; Copernicus deems this fictive point an inelegant attempt at "saving the phenomena"

imāginārentur: imperfect subjunctive in a mixed present contrary-to-fact conditional; whereas the subject of *sufficiēbant* is still the implied antecedent of *quae*, the implicit subject of *imāginārentur* is "Ptolemy and very many others everywhere"

quibus: ablative of means and relative pronoun whose antecedent is *aequantēs quōsdam circulōs*

Quāpropter nōn satis absolūta vidēbātur huiusmodī speculātiō neque ratiōnī satis concinna. Igitur cum haec animadvertissem ego, saepe cōgitābam, sī forte ratiōnābilior modus circulōrum invenīrī possit, ⇒

---

quāpropter: for which reason, therefore
satis: (as adv.) sufficiently
speculātiō, -ōnis f.: investigation; theory (cf. speculation)
ratiō, -ōnis f.: reason, logic
concinnus, -a, -um: beautiful, elegant
igitur: therefore (normally the second word of a sentence)
cōgitō (1): think, ponder
forte (adv.): by chance
ratiōnābilis, -e: reasonable, suitable
modus, -ī m.: (here) form, pattern
inveniō, -īre, -vēnī, -ventus: find

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nōn satis absolūta: as defined earlier, *absolūta* means "perfect" or "complete" in this context; the Ptolemaic model is *ad hoc* and always in a certain sense "incomplete" because, if there are ever discrepancies between the observations and the predictions, more epicycles can be added (hence the expression "adding epicycles"), not to mention that the equant serves no purpose other than to restore the principle of uniform motion; unfortunately, although Copernicus was able to remove some of Ptolemy's epicycles in his model by centering the orbits at the sun, he was compelled to add epicycles of his own to remove the equant entirely (because, as it turns out, the planetary orbits are not circular but elliptical)

huiusmodī: *of this kind*; a common merging of *huius* and *modī*

cum . . . animadvertissem ego: circumstantial *cum* clause with a pluperfect subjunctive verb

sī: *whether*, introducing an indirect question with a subjunctive verb; a weaker alternative to the slightly more Classical *num*

(continued) ē quibus omnis appārēns dīversitās dēpendēret, omnibus in sē ipsīs aequāliter mōtīs, quemadmodum ratiō absolūtī mōtūs poscit.

---

dīversitās, -ātis f.: diversity, variety
dēpendeō, -ēre, –, –: depend on, derive from
quemadmodum: in which manner; just as
poscō, -ere, poposcī, –: require

ē quibus . . . dēpendēret: the antecedent of *quibus* is the plural *circulōrum*; the verb is subjunctive as the relative clause is expressing a hypothetical idea (and is imperfect because the sentence's main verb, *cōgitābam*, is in the past tense)

omnibus in sē ipsīs aequāliter mōtīs: ablative absolute with subject *omnibus* and PPP *mōtīs*; note that, as a verb of motion, *mōtīs* can be translated (intransitively) as a perfect active participle; "moving in themselves" means that all circles should move at a uniform speed from the perspective of their centers, and not from that of an artificial equant point

ratiō absolūtī mōtūs: *the law of perfect* — i.e. both circular and uniform — *motion*

Rem sānē difficilem aggressus ac pēne inexplicābilem obtulit sē tandem, quōmodo id pauciōribus ac multō convenientiōribus rēbus quam ōlim sit prōditum fierī possit, sī nōbīs aliquae petītiōnēs, quās axiōmata vocant, concēdantur: quae hōc ōrdine sequuntur.

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sānē (adv.): (here, with adj.) very
aggressus, -ūs m.: attack; approach
pēne (adv.): nearly, almost
inexplicābilis, -e: (referring to a problem) unsolvable
offerō, -ferre, obtulī, oblātus: show, present
tandem: at last, finally, in the end
quōmodo: in which way
multō (adv.): much
conveniēns, -entis: appropriate, consistent, harmonious
ōlim (adv.): once, some time ago, in the past
petītiō, -ōnis f.: (here) postulate
axiōma, -atis n.: axiom (from the Greek ἀξίωμα, something "worthy" of being assumed)
concēdō, -ere, -cessī, -cessus: assent, concede
ōrdō, -inis m.: order

Rem sānē difficilem aggressus ac pēne inexplicābilem obtulit sē: *an approach to the very difficult and almost unsolvable matter presented itself*; *aggressus* takes an accusative object as if it were a participle

quōmodo: introduces an indirect question with the subjunctive verb *possit*

quam ōlim sit prōditum: *than was once put forth*; the comparison with *pauciōribus* and *convenientiōribus* uses *quam*; note that *sit prōditum* is subjunctive because it is also dependent on the indirect question

sī . . . concēdantur: a mixed future-less-vivid conditional with a present subjunctive verb usually translated with "should"; inspired by the axiomatic treatment of geometry in Euclid's *Elements*, mathematicians sought to introduce postulates to other domains; notably, in his mechanistic philosophy, René Descartes wanted to deduce *ā priōrī* all physical phenomena from the simple interactions of fundamental (infinitely divisible) particles known as corpuscles

quae: this relative pronoun, which has *petītiōnēs* as its antecedent, can be taken as a demonstrative pronoun

hōc ōrdine: *in the following order*, where *hōc* refers to what comes next

Prīma petītiō

Omnium orbium coelestium sīve sphaerārum ūnum centrum nōn esse.

Secunda petītiō

Centrum Terrae nōn esse centrum mundī, sed tantum gravitātis et orbis Lūnāris.

Tertia petītiō

Omnēs orbēs ambīre Sōlem tamquam in mediō omnium existentem, ideōque circā Sōlem esse centrum mundī.

---

sīve: or

sphaera, -ae f.: sphere (from the Greek σφαῖρα)

mundus, -ī m.: world, universe (cf. French "monde")

tantum: only

gravitās, -ātis f.: weight; attractive force to the Earth, gravity

lūnāris, -e: lunar

ambiō, -īre, -īvī, -ītus: go around

tamquam: (here, with a participle) as if it were

medium, -iī n.: middle

existō = exsistō, -ere, -stitī, –: exist; be situated

ideō (adv.): for that reason, therefore (from *id* + *eō*, lit. "this from that")

Prīma petītiō: there has been some debate as to whether these "postulates" qualify as such, primarily because some of them overlap or technically follow from other postulates (e.g. the first can be derived from the second and third); nevertheless, each of them helps emphasize an important feature of the Copernican system

Omnium orbium coelestium . . . nōn esse: all the postulates are structured as indirect statements (as is common throughout scientific Latin works) and can be translated as a sentence introduced by "that"

Terrae . . . mundī: note the distinction between the "Earth" and the "universe"

Quārta petītiō

Minōrem esse comparātiōnem distantiārum Sōlis et Terrae ad altitūdinem firmāmentī, quam sēmidīmētientis Terrae ad distantiam Sōlis, adeō ut sit ad summitātem firmāmentī īnsēnsibilis.

---

comparātiō, -ōnis f.: comparison, relation; (with gen. and *ad*) ratio of (something) to (something)

distantia, -ae f.: distance

altitūdō, -inis f.: height, altitude

firmāmentum, -ī n.: (here, describing the outermost sphere of the stars) firmament

sēmidīmētiēns, -entis f.: semidiameter, radius

adeō (adv.): so far, to such a degree

summitās, -ātis f.: highest part, height

īnsēnsibilis, -e: insensible; imperceptible

Minōrem . . . quam: the comparison uses *quam*

sēmidīmētientis . . . ad distantiam: these words are dependent on their own implicit *comparātiōnem*

adeō ut: *to the point that*; *adeō* anticipates a result clause

sit: the subject is the distance between the Sun and the Earth; Copernicus specifies that the diameter of the Earth's orbit is far smaller than the diameter of the universe because, otherwise, stellar parallax (the apparent movement of the stars in the night sky due to the motion of the Earth) would be very noticeable to the human eye; the latter was one of the arguments in favor of a geocentric model

Quīnta petītiō

Quicquid ex mōtū appāret in firmāmentō nōn esse ex parte ipsīus, sed Terrae. Terra igitur cum proximīs elementīs mōtū diurnō tōta convertitur in polīs suīs invariābilibus firmāmentō immōbilī permanente ac ultimō coelō.

---

quīntus, -a, -um: fifth
quisquis, quidquid (quicquid): whoever, whatever
proximus, -a, -um: nearest
elementum, -ī n.: (here) element, body
diurnus, -a, -um: daily
tōtus, -a, -um: (used as a quasi-adverb) wholly; in his/her/its entirety
convertō, -ere, -ī, -versus: turn round, rotate
polus, -ī m.: pole
invariābilis, -e: invariable, fixed
immōbilis, -e: immobile, motionless
permaneō, -ēre, -mānsī, -mānsus: remain
ultimus, -a, -um: (here) most distant
coelum = caelum

---

nōn . . . ex parte ipsīus, sed Terrae: *not on the part of* (or *"due to"*) *itself but the Earth*, paralleling the phrasing of the next postulate

mōtū diurnō: ablative of means

convertitur: a passive verb of motion with an active, intransitive meaning

in polīs suīs invariābilibus: in all geocentric models, the sphere of the stars revolves, but Copernicus ascribes this apparent movement to the Earth's rotation around a fixed axis; since Copernicus, scientists have found that the Earth's axis does change slightly over time and can be influenced by human activity (e.g. the creation of dams)

firmāmentō immōbilī permanente ac ultimō coelō: an ablative absolute with the PAP *permanente* and the nouns *firmāmentō* and *ultimō coelō*

Sexta petītiō

Quicquid nōbīs ex mōtibus circā Sōlem appāret nōn esse occāsiōne ipsīus, sed Tellūris et nostrī orbis, cum quō circumvolvimur ceu aliquō aliō sīdere, sīcque Terram plūribus mōtibus ferrī.

Septima petītiō

Quod appāret in errāticīs retrōcessiō ac prōgressus nōn esse ex parte ipsārum, sed Tellūris. Huius igitur sōlīus mōtus tot appārentibus in coelō dīversitātibus sufficit.

---

sextus, -a, -um: sixth

occāsiōne (+ gen.): on account of (abl. of *occāsiō, -ōnis f.*, "occasion")

tellūs, -ūris f.: Earth (cf. tellurium)

circumvolvō, -ere, -ī, -volūtus: revolve around

ceu: like, as

plūs, plūris: more; (pl.) several (comparative of *multus, -a, -um*; in the singular, usually only neuter)

septimus, -a, -um: seventh

errāticus, -a, -um: wandering; planet (as a feminine substantive and calque of the Ancient Greek πλάνητες ἀστέρες, "wandering stars")

retrōcessiō, -ōnis f.: a going backward; (in astronomy) retrograde motion

prōgressus, -ūs m.: a going forward; (in astronomy) prograde motion

tot (indecl.): so many

---

ex mōtibus circā Sōlem: *from the motions concerning the Sun*; Copernicus seems to intentionally distance any mention of the Sun from any actual (i.e. not apparent) motions

Tellūris et nostrī orbis: Copernicus distinguishes between the axial rotation of the Earth itself and its circumsolar motion caused by its celestial orb or sphere

cum quō . . . aliquō aliō sīdere: both are ablatives of accompaniment, as we humans move along or "with" the surface of the Earth and the celestial sphere during their rotation; the antecedent of *quō* is *Tellūris* and *nostrī orbis*

volvimur: like *ferrī* below, *volvimur* is a passive verb of motion with an active, intransitive meaning

Terram plūribus mōtibus ferrī: *plūribus mōtibus* is an ablative of means; *ferrī* is a second infinitive of the indirect statement

Quod: its implicit antecedent *id* is the subject of *esse*

retrōcessiō ac prōgressus: when a planet is on the same side of the Sun as the Earth, it appears to reverse its course in the night sky and to undergo "retrograde motion"; the epicycles in the Ptolemaic model were an ancient explanation of this phenomenon (see Figure 5)

Huius . . . sōlīus: *of this (Earth) alone*; refers to *Tellūris*

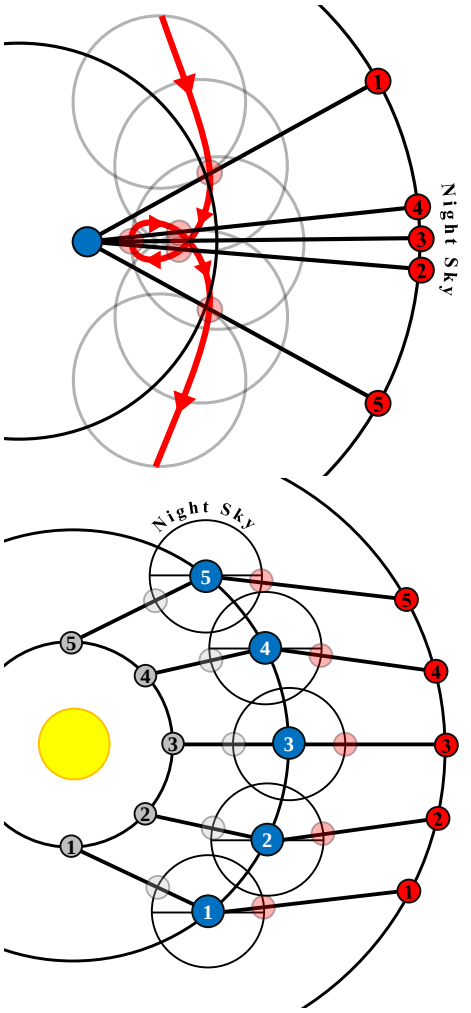


Figure 5: In the Ptolemaic system (top), retrograde motion occurs when the rotation of the epicycle (grey) counters that of the deferent. When the path of the planet (red) is traced out, the loops correspond to these periods of retrograde motion. In the Copernican system (bottom), the appearance of retrograde motion has a more direct cause, the Earth (blue) overtaking or being overtaken by another planet (grey or red) in its orbit around the sun.

Hīs igitur sīc praemissīs cōnābor breviter ostendere quam ōrdinātē aequālitās mōtuum servārī possit. Hīc autem brevitātis causā mathēmaticās dēmōnstrātiōnēs omittendās arbitrātus sum, maiōrī volūminī dēstinātās.

---

praemittō, -ere, -mīsī, -missus: describe first, set forth (in a previous passage); (as a feminine substantive PPP) premise

cōnor, -ārī, -ātus sum: try, attempt

breviter (adv.): briefly

quam: (here) how

ōrdinātē (adv.): in order; properly, correctly

aequālitās, -ātis f.: equality, uniformity

brevitās, -ātis f.: brevity

mathēmaticus, -a, -um: mathematical (from the Greek μαθηματικός, ultimately from μανθάνειν, "to learn")

dēmōnstrātiō, -ōnis f.: demonstration, proof

omittō, -ere, -mīsī, -missus: pass over, omit

arbitror, -ārī, -ātus sum: judge, decide

volūmen, volūminis n.: roll of writing; book, volume

dēstinō (1): (here, with dat.) devote to, dedicate to; intend for

---

Hīs . . . praemissīs: *with these (postulates) having been set forth* or *by means of these premises*; can be taken as either an ablative absolute with PPP *praemissīs* or an ablative of means

quam . . . possit: indirect question with a subjunctive verb

Hīc: *here*, in contrast to the *maiōrī volūminī*, which very likely describes his eventual *Dē Revolūtiōnibus*

brevitātis causā: *for the sake of brevity*

mathēmaticās dēmōnstrātiōnēs omittendās: an indirect statement introduced by *arbitrātus sum*; assume an *esse* to complete the passive periphrastic infinitive *omittendās esse*

dēstinātās: modifies *mathēmaticās dēmōnstrātiōnēs*

Quantitātēs tamen sēmidiametrōrum orbium in circulōrum ipsōrum explānātiōne hīc pōnentur, ē quibus mathēmaticae artis nōn ignārus facile percipiet quam optimē numerīs et observātiōnibus tālis circulōrum compositiō conveniat.

---

quantitās, -ātis f.: amount; quantity; magnitude
sēmidiametros, -ī m.: semidiameter, radius
explānātiō, -ōnis f.: explanation
ars, artis f.: art
ignārus, -a, -um (+ gen.): ignorant (of)
facile (adv.): easily (adverbial form of *facilis, -e*)
percipiō, -ere, -cēpī, -ceptus: perceive, understand
optimē (adv.): best, very well (comparative of *benē (adv.)*)
observātiō, -ōnis f.: observation
tālis, -e: such

ē quibus: its antecedent is *circulōrum ipsōrum*

nōn ignārus: *a person not ignorant of*; this negation of a substantive adjective is an example of litotes

quam . . . conveniat: another indirect question that requires the subjunctive

Proinde nē quis temere mōbilitātem Tellūris assevērāsse cum Pȳthagoricīs nōs arbitrētur, magnum quoque et hīc argūmentum accipiet in circulōrum dēclārātiōne.

---

proinde (adv.): hence, accordingly
temere (adv.): (here) carelessly, without thought
mōbilitās, -ātis f.: mobility
assevērō (1): affirm
Pȳthagoricus, -ī m.: a Pythagorean (Latinization of Πυθαγορικός; see below)
argūmentum, -ī n.: argument
dēclārātiō, -ōnis f.: declaration; explanation

Nē . . . arbitrētur: negative purpose clause, often introduced by "lest"

quis: after *nē*, *quis* is equivalent to *aliquis*; this imagined objector is also the subject of *accipiet* later in the sentence

assevērāsse: syncopation of the perfect active infinitive *assevērāvisse*

cum Pȳthagoricīs: *in agreement with the Pythagorians*; an ablative of accompaniment; the 5th-century BCE philosopher and Pythagorean Philolaus is thought to have suggested that all planets, including the Earth and the Sun, rotated around a "Central Fire" always on the opposite side of Greece. Perhaps to satisfy their desire to have exactly 10 celestial bodies, the Pythagoreans also believed in an Ἀντίχθων or "Counter-Earth" orbiting diametrically opposite to the Earth. Kepler would later argue that the Central Fire and the Sun were actually one and the same, as were the Counter-Earth and the Moon (Granada, 2022). Other Ancient Greek astronomers who conceived of non-geocentric models were Aristarchus of Samos (with the first true heliocentric system) and possibly Heraclides of Pontus (with a mixed system where only Mercury and Venus orbit the sun instead of the Earth)

Etenim quibus physiologī stabilitātem eius astruere potissimē cōnantur, appārentiīs plērumque innītuntur; quae omnia hīc inprīmīs corruunt, cum etiam propter appārentiam versēmus eandem.

---

etenim: for (usually introducing a self-evident statement)
physiologus, -ī m.: natural philosopher (from the Greek φύσις, "nature")
stabilitās, -ātis f.: stability
astruō, -ere, -strūxī, -strūctus: affirm; demonstrate, prove
potissimē (adv.): chiefly, especially, above all
appārentia, -ae f.: appearance
plērumque (adv.): for the most part, mostly
innītor, -ī, -nīxus sum (+ dat.): (here) rely on
inprīmīs (adv.): especially; first of all; in the first place
corruō, -ere, -ī, -itūrus: fall down, collapse
propter (+ acc.): (here) by means of
versō (1): (here) turn upside down

---

quibus physiologī . . . cōnantur: a relative clause with *quibus* being an ablative of means; the implicit antecedent of *quibus* (the arguments that philosophers use to defend geocentrism) is the subject of *innītuntur*

quae omnia: *omnia* is modifying the relative pronoun, the antecedent of which is the (neuter) subject of the previous clause

cum etiam propter appārentiam versēmus eandem: *since we also turn the same (stability) upside down by means of appearance*; causal *cum* clause with a present subjunctive verb; *eandem* refers to *stabilitātem* from the previous sentence; Copernicus attributes the apparent immobility of the Earth to the fact that we as observers move alongside it (as an example of relativity)

Chapter 4 — Kepler: *Somnium*

Johannes Kepler (1571 CE – 1630 CE) was a fervent supporter of heliocentrism ever since he encountered Copernicus's work during his education at the University of Tübingen. However, while his primary interest was astronomy, he practiced astrology as his profession. A recurring objective in his texts was to find God's vision for the universe, as embodied in his geometric *ā priōrī* explanations for the structure of the Copernican system. His first attempt, laid out in the *Mystērium Cosmographicum*, required an elegant arrangement of Platonic solids: The innermost sphere centered at the Sun defines the size of Mercury's orbit and is circumscribed by an octahedron, which in turn is circumscribed by the sphere for Venus. Continuing with an icosahedron, dodecahedron, tetrahedron, and finally a cube, Kepler was able to approximate the ratios between the distances of the planets to the sun with some accuracy. Furthermore, since only five Platonic solids exist (as proven in Book 13 of Euclid's *Elements*, the foundational textbook for geometry), this construction accounted for the existence of the six planets known at the time (Di Liscia, 2025).

Much to his chagrin, Kepler had to reject the model in his 1609 *Astronomia Nova*, once he realized that the planets actually moved elliptically. In publishing this discovery, he upended a principle that had defined astronomy for nearly two millennia, namely that the planetary orbits could be described as compositions of circles. As he acknowledged in the work's full title,[1] this finding was heavily indebted to the data reluctantly provided by Tycho Brahe from his

[1]It reads *Astronomia Nova* Αἰτιολόγητος*, seu physica coelestis, trādita commentāriīs dē mōtibus stēllae Mārtis ex observātiōnibus G.V. Tychōnis Brahe*, "A new causative astronomy, or a celestial physics, delivered in commentaries about the motions of the planet Mars from the observations of Your Excellence Tycho Brahe." αἰτιολόγητος, a Greek compound for "pertaining to the investigation of causes," once again hints at Kepler's *ā priōrī* reasoning.

state-of-the-art observatory (see Figure 1). Tycho had originally constructed it in an attempt to distinguish between the older, geocentric Ptolemaic system and the Copernican model by measuring the parallax of Mars. This measurement led him to conclude that Mars could periodically get closer to the Earth than the Earth to the Sun, which would only occur in Copernicanism. However, instead of accepting the Copernican system outright, Tycho ultimately constructed an equivalent geocentric model through a change of reference frame that is equally consistent with his measurement. For this reason, it turns out to be impossible to establish by observation alone whether the Earth orbits the Sun or vice-versa, revealing a limitation of *ā posteriōrī* verifications that Tycho relied upon. In contrast, Kepler favored an *ā priōrī* approach (Di Liscia, 2025), in which there exists either a cosmological reason explaining why the universe is as it is (as was the case in his Platonic solid model mentioned above), or a derivation of natural phenomena from simple, universal laws (Gingerich and Voelkel, 2005), such as his three ground-breaking laws of planetary motion (see Figure 2). Newton later showed that Kepler's laws could be reduced to his own, more fundamental, laws of motion.

Given Kepler's prominence in the academic debates of his time, it may come as a surprise that he also tried popularizing his scientific discoveries in the form of a fictional "dream" involving a journey to the Moon (Kepler, 1634). This story, the *Somnium*,[2] which had been drafted in 1608 but was only published posthumously in 1634, initially follows a boy named Duracotus, who returns from an apprenticeship in astronomy under the supervision of — uncoincidentally — Tycho Brahe to his mother Fiolxhildis. She then introduces him

[2]Full Latin title: *Somnium,* Seu *Opus Posthumum Dē Astronomiā Lūnārī*

to the daemons of the Moon, summoning one of them to explain to them how a human could travel to the heavens with explosives, momentum, and a bit of spectral assistance. The daemon even discusses lunar astronomy, where the roles of the Earth and Moon are swapped but the laws of physics remain unchanged, in an instance of Galilean invariance (this principle allowed Tycho's aforementioned repositioning of the center of the universe in the Copernican model). Indeed, from the Moon, the Earth appears to be an orbiting satellite with lunar phases, hence its name Volva (cf. re*volve*) among the daemons. Kepler supplements his tale with over two hundred footnotes full of rigorous scientific explanations (and humor). Thanks to this level of care in the physics, the *Somnium* is often hailed as the first work of "hard" science fiction, even though, as Kepler himself admits, his story shares many elements with Classical literature, including: the dream narrative device that is also present in Cicero's aptly-named *Somnium Scīpiōnis*; the setting of Iceland, which corresponds to the ancient mythical island of Thule in the North; and last but not least the voyage to the Moon itself (Christianson, 1976). Kepler claims to have been inspired by both Lucian's *Vēra Historia*, which very much is not a "True Story" and instead mocks the natural philosophy of his contemporaries (Boissoneault, 2016), and by Plutarch's *Dē faciē quae in orbe lūnae appāret*, which debates the science behind the "Face in the Moon" and even proposes a kind of Cartesian mind-matter dualism (Gárriga, 2023).

One particularly prescient footnote in the excerpt below, number 62, investigates the causes of the tides on Earth. Philosophers in ancient times generally agreed that the ebb and flow of the ocean, both in their duration and intensity, depended on the motions and phases of the Moon, although some, like Aristotle (who according to

legend became so frustrated with the unpredictable tides of the Euripus straight that he drowned himself in it), instead believed that the Sun and its warmth induced the displacement of the water through winds (Tabarroni, 1989). In the footnotes numbered 66, 74, 75, and 78 below, Kepler conceives of a gravitational attraction not only to the Earth, as Aristotelian physics would contend, but to any massive celestial body like the Moon. He then correctly attributes the tides to the tension between such forces exerted by the Earth and Moon. Indeed, Kepler explains, in a way a modern schoolchild may understand, that the water directly underneath the Moon rises towards it due to gravitational attraction, causing tidal flow, but, as the Moon moves away, the ocean ebbs as it falls back towards the Earth. Admittedly, Kepler doesn't provide a quantitative model for the tides as Newton eventually accomplished, and he makes no mention of antipodal tides (caused by the Earth being attracted towards the Moon more strongly than the oceans opposite to the Moon). Still, Kepler's treatment is far superior to Galileo Galilei's infamous claim that the tides are the result of the oceans being sloshed around by the moving Earth, which directly conflicts with the idea that the Earth's rotation around the Sun is imperceptible to humans on its surface. Although Galileo's other studies were, ironically, devoted to demonstrating the latter assertion, he chose to frame his theory of the tides as a definite proof for heliocentrism in his *Dialogue Concerning the Two Chief World Systems*, not realizing the contradictions that ensue. Galileo fixated on having found a mechanistic cause for the tides and explicitly criticized Kepler's approach to the tides for requiring an apparently unphysical long-distance interaction between the Earth's waters and the Moon (Machamer and Miller, 2021), a charge later directed towards Newton for his theory of universal gravitation.

Source Text: Our Latin text comes from Kepler (1634).

Note: The superscripts interspersed throughout the *Somnium* point to Kepler's own footnotes. Due to length considerations, only a subset of them are reproduced here after the main excerpt. Unlike modern conventions, the superscript *precedes* the relevant section.

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Figure 1: Tycho Brahe measuring the altitude of stars using a quadrant at his Uraniborg observatory, which was named after the Greek god of the heavens. Image credit: Utrecht University Library, public domain.

An Overview of Kepler's Three Laws

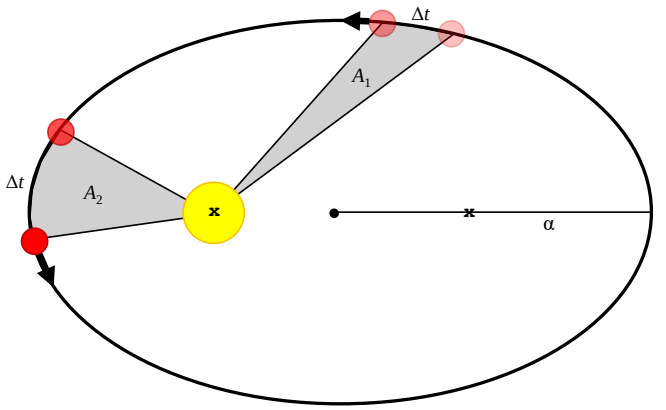


Figure 2: Kepler's three laws of planetary motion are:

- that the orbit of a planet is an ellipse, with the Sun as one of the foci.
- that, if the lines are drawn from the center of a planet to the center of the Sun at times t and $t + \Delta t$, the area contained within these two lines and the elliptical orbit is a constant dependent on Δt only and not t (so the areas denoted A_1 and A_2 are equal). The second law was actually the one Kepler developed first and, as Newton later showed, is the most general, only requiring an attractive force of arbitrary strength towards a fixed point like the sun. The law implies that planets accelerate as they approach the Sun (earlier astronomers had insisted that they should move at a constant speed).
- that the period of a planet's orbit is proportional to $\alpha^{\frac{3}{2}}$, where α is the semimajor axis of the ellipse. Unlike the other laws, which had been published in the *Astronomia Nova*, Kepler presented the third law in the 1619 *Harmonicē Mundī*, where he had to initially guess the exponent that would lead to such a relationship through a copious amount of data. He later provided a mathematical justification for it in the *Epitomē Astronomiae Copernicānae* (Cohen, 1985).

Somnium, *Seu* Opus Posthumum Dē Astronomiā Lūnārī Daemōn ex Levāniā

[53]Quīnquāgintā mīllibus mīliārium Germānicōrum in aetheris profundō [54]sita est LEVĀNIA īnsula: [55]iter ad eam hinc, vel ex eā in hās terrās rārissimē patet: & cum patet [56]nostrae quidem gentī facile est; ⇒

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somnium, -iī n.: dream
seu: or
opus, operis n.: work
posthumus = postumus, -a, -um: posthumous (superlative of *posterus, -a, -um*, literally referring to a text published "very late"; the word later became associated with *humus, -ī f.*, the "ground" in which an author is buried)
Astronomia, -ae f.: astronomy
lūnāris, -e: of the Moon, lunar
daemōn, -onis m.: spirit, demon
Levānia, -ae f.: the name of Kepler's land on the Moon, inspired by the Hebrew word for "Moon" (לְבָנָה *levānāh*, the feminine form of לָבָן *lāvān*, "white")
quīnquāgintā (indecl.): fifty
mīliāre, -is n. = mīliārium, -iī n.: mile-stone; mile
Germānicus, -a, -um: German
aethēr, -eris m.: heaven (from the Greek αἰθήρ, the element of the heavens)
profundum, -ī n.: depth (substantive form of *profundus, -a, -um*, "deep")
situs, -a, -um: (here, with *esse*) lie, be situated
iter, itineris n.: journey
hinc: from this place
rārissimē: very rarely (adverbial superlative form of *rārus, -a, -um*)
pateō, -ēre, -uī, –: lie open
gēns, gentis f.: kind, people

Astronomia: in the *Epitomē*, Kepler argues that, because the name is derived from the Greek words ἄστρον, "star," and νόμος, "law" (like economy, from οἶκος, "house," and νόμος), astronomy should be devoted to not only planetary motions but also the laws that govern them

Quīnquāgintā mīllibus mīliārium Germānicōrum: *fifty thousand (of) German miles*; *quīnquāgintā mīllibus* is an ablative of degree of difference; *mīlle* takes a genitive, a grammatical construction that is partially present in English (e.g. "thousands of men"); a (geographical) German mile is approximately equal to 7,420 m, so the distance between the Earth and the Moon specified here is roughly 371,000 km (which is close to the actual distance of 384,400 km)

nostrae . . . gentī: the speaker is one of the lunar daemons that introduces the protagonist and the reader to its homeworld

(continued) [57]hominibus vērō trānsportandīs plānē difficilimum, & cum summō vītae perīculō coniūnctum: [58]Nūllī ā nōbīs sedentāriī adscīscuntur in hunc comitātum, nūllī corpulentī, nūllī dēlicātī; [59]sed legimus eōs quī aetātem verēdōrum assiduō ūsū cōnsūmunt, aut quī nāvibus frequenter Indiās adeunt, pāne biscoctō, alliō, piscibus dūrātīs, & cibīs abhorrentibus vīctitāre suētī.

---

vērō (adv.): in truth; but, and (post-positive)
trānsportō (1): carry over, transport
plānē (adv.): clearly
perīculum, -ī n.: danger
coniungō, -ere, -iūnxī, -iūnctus (+ *cum*): bind together with, connect with
sedentārius, -a, -um: sedentary
adscīscō = ascīscō, -ere, -scīvī, -scītus: receive, approve
comitātus, -ūs m.: retinue, company (here, the party of travelers)
corpulentus, -a, -um: corpulent, fat
dēlicātus, -a, -um: addicted to pleasure; delicate
legō, -ere, lēgī, lēctus: (here) select
aetās, -ātis f.: lifetime, life
verēdus, -ī m.: fast, light horse
assiduus, -a, -um: constant, frequent
ūsus, -ūs m.: use
cōnsūmō, -ere, -psī, -ptus: consume, (with respect to time) spend
nāvis, -is m.: ship
frequenter: frequently
India, -ae f.: India; (pl.) the Indies
adeō, -īre, -iī, -itus: go to
pānis, -is m.: bread
biscoctus, -a, -um: twice-cooked
allium = ālium, -iī n.: garlic
piscis, -is m.: fish
dūrō (1): harden (here describing preserved food)
cibus, -ī m.: food
abhorrēns, -entis: unusual; repulsing
vīctitō (1): subsist
suēscō, -ere, suēvī, suētus: be accustomed

---

hominibus . . . trānsportandīs: *for transporting humans*; a gerundive phrase that can be rewritten as the gerund construction *hominēs trānsportandō* (where the gerund is a dative of reference)

hominibus . . . difficilimum, & . . . coniūnctum: assume an *est* in both main clauses; *coniūnctum est* should be translated as *est* with a PPP (i.e. "it is connected with" instead of the perfect passive indicative "it was connected with"), as is quite common in post-Classical works; also in later Latin, double letters could sometimes be confused with single letters, hence *difficilimum* for *difficillimum*

assiduō ūsū . . . nāvibus: ablatives of means

Indiās: *to the Indies*; a preposition is not necessary for names of towns, islands, or, in this case, a collection of small isles

pāne biscoctō, . . . & cibīs abhorrentibus: ablative of means with *vīctitāre* ("to subsist by means of" or "on" various kinds of supplies)

[60]Inprīmīs nōbīs aptae sunt vetulae exsuccae, quibus inde ā pueritiā trīta est ratiō, hircōs nocturnōs, aut furcās, aut trīta pallia inequitandī, trāiiciendīque per immānia terrārum spacia; [61]Nūllī ē Germāniā virī aptī sunt: Hispānōrum sicca corpora nōn respuimus.

---

inprīmīs (adv.): especially
aptus, -a, -um: suitable
vetulus, -a, -um: little old (diminutive of *vetus, veteris*)
exsuccus = exsūcus, -a, -um: without juice, dried out
inde: since; since the very beginning of (when emphasized with *ab*)
pueritia, -ae f.: boyhood, childhood
trītus, -a, -um: common, familiar (cf. trite; see *terō* below)
ratiō, -ōnis f.: (here) manner, habit
hircus, -ī m.: goat
nocturnus, -a, -um: nocturnal; (here) by night
furca, -ae f.: fork; (here) pitchfork
terō, -ere, trīvī, trītus: rub, wear out
pallium, -iī n.: cloak
inequitō (1): ride on
trāiiciō = trāiciō, -ere, -iēcī, -iectus: pass through
immānis, -e: immense, vast
spacium = spatium, -iī n.: space, span
Germānia, -ae f.: Germany
Hispānus, -ī m.: Spaniard
siccus, -a, -um: dry; firm
respuō, -ere, -ī, –: reject

---

vetulae exsuccae: *little old, dry women*, or, of course, witches; when Kepler's father left the family to enlist as a solider, his mother made a living by producing and selling pharmaceutical herbs, leading to a witch trial against her from 1611 to 1621. The mother of the protagonist Duracotus is herself a witch, summoning the daemons of Levania. An early version of the *Somnium* that may have been circulated in Kepler's hometown of Tübingen at the time could have possibly contributed to the accusation against his mother (American Physical Society, 2015)

quibus . . . trīta est ratiō: *who have a familiar habit*; a dative of possession construction, which can be rephrased as *quae . . . trītam ratiōnem habent*; *trīta* modifies *ratiō* and suggests either that this habit is common (i.e. the witches frequently ride goats and pitchforks), or that the reader is very familiar with it (i.e. Kepler is alluding to the archetypal activities of witches)

hircōs nocturnōs . . . inequitandī: gerund construction with three accusative objects; *inequitandī* and *trāiiciendī* afterwards are objective genitives governed by *ratiō* (i.e. "habit of riding . . . and passing through . . .")

[62]Tōtum iter quantum est, quātuor ad summum hōrārum spaciō absolvitur. [63]Neque enim nōbīs semper occupātissimīs, anteā cōnstat dē tempore eundī, quam Lūna ab orientis partibus cēperit dēficere: quae ubi tōta lūxerit, nōbīs adhūc in itinere haerentibus: irrita redditur nostra profectiō.

---

quantus, -a, -um: however great
quātuor = quattuor
hōra, -ae f.: hour
absolvō, -ere, -ī, -solūtus: release; (here) complete
occupātus, -a, -um: occupied, busy
anteā (adv.): before (lit., with *quam*, "earlier than")
cōnstō, -āre, -stitī, -stātūrus: stand still; (here impersonally with *dē*) it is settled with respect to, (idiomatically) be settled
Lūna, -ae f.: the Moon
oriēns, -entis m.: the East (in reference to the "rising" sun)
cēpī = coepī, -isse, -tus: begin
dēficiō, -ere, -fēcī, -fectus: be wanting; (here, in an astronomical context) be eclipsed
tōtus, -a, -um: (used as a quasi-adverb) wholly; in his/her/its entirety
lūceō, -ēre, lūxī, –: shine
adhūc (adv.): (here) until now, still
haereō, -ēre, haesī, haesus (+ *in*): adhere, stick to, fix in
irritus, -a, -um: void, of no effect
reddō, -ere, -didī, -ditus: return; (here) render
profectiō, -ōnis f.: a setting out, departure

---

quātuor . . . hōrārum spaciō: *spaciō* is an ablative of time within which governing the objective genitive *quātuor . . . hōrārum*

ad summum: *at most*

Neque enim: this phrase is the negation of *etenim*, both of which are used to indicate that a statement is self-evident (*neque enim* is translatable as "for clearly . . . not" or "for of course . . . not")

nōbīs semper occupātissimīs: *for us, (who are) always very busy*

tempore eundī: *time of going* or *departure*

quae: relative pronouns are often used as demonstrative pronouns; in this case, *quae* can be translated as "it" or "this", referring to *Lūna*; a connective to a previous clause or sentence can precede a conjunction like *ubi* even if it belongs to the subsequent clause

nōbīs adhūc in itinere haerentibus: ablative absolute with *nōbīs* as the noun and *harentibus* as the PAP

[64]Tam praeceps occāsiō efficit ut paucōs ex hūmānā gente, nec aliōs, nisi nostrī observantissimōs comitēs habeāmus. [65]Ergō hominem aliquem huius modī agminātim invādimus, omnēsque subtus nītentēs, in altum eum tollimus. [66]Prīma quaeque mōlītiō dūrissima ipsī accidit.

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praeceps, -cipitis: headlong; (here) fleeting
occāsiō, -ōnis f.: occasion, opportunity
efficiō, -ere, -fēcī, -fectus: work out; (here) produce, bring about
hūmānus, -a, -um: human
observāns, -antis (+ gen.): respectful of (someone)
comes, comitis m./f.: companion
agminātim (adv.): by troops
invādō, -ere, -vāsī, -vāsus: enter violently, assail; seize
subtus (adv.): below
nītor, -ī, nīxus sum: bear; advance; strive, labor
tollō, -ere, sustulī, sublātus: lift, raise
mōlītiō, -ōnis f.: a putting in motion, launch
ipse, -a, -um: (here, simply as a pronoun) he, she, it
accidō, -ere, -ī, – (+ dat.): fall upon, hit

---

ut paucōs . . . habeāmus: a noun clause functioning as the object of *efficit*
nostrī: an objective genitive governed by *observantissimōs*
in altum: an idiom for "on high" or "to the sky"
Prīma quaeque: an expression for "the very first"

[67]Nec enim aliter torquētur ac sī pulvere Bombardicō excussus montēs & maria trānāret. [68]Proptereā Narcōticīs & Opiātīs statim in prīncipiō sōpiendus est, & [69]membrātim explicandus, nē corpus ā pōdice, caput ā corpore gestētur, sed ut violentia in singula membra dīvidātur.

---

aliter (adv.): otherwise
torqueō, -ēre, torsī, tortus: twist; launch (used in Classical Latin both for preparing a shot with a spear and for winding up a *tormentum*, a type of catapult)
ac: (here, in a comparison) as, than
pulvis, pulveris m.: powder
bombardicus, -a, -um: pertaining to guns
excutiō, -ere, -cussī, -cussus: shake out, send forth
trānō (1): swim, fly over
proptereā (adv.): therefore
narcōticum, -ī n.: narcotic
opiāta, -ae f.: opiate
statim (adv.): at once
prīncipium, -iī n.: beginning
sōpiō, -īre, -īvī, -ītus: deprive of feeling, make unconscious
membrātim (adv.): limb by limb
explicō (1): unfold
pōdex, pōdicis m.: butt
gestō (1): carry off
violentia, -ae f.: violence
singulī, -ae, -a: individual
membrum, -ī n.: limb
dīvidō, -ere, -vīsī, -vīsus: divide; distribute

Nec . . . aliter . . . ac sī: *not otherwise than if* or, more directly, *just as if*

pulvere Bombardicō excussus: a participial phrase modifying the implied subject "he" of *torquētur* and *trānāret*; *pulvere Bombardicō* is an ablative of means

trānāret: an imperfect subjunctive verb in the present contrary-to-fact protasis of a mixed conditional

sōpiendus est, . . . explicandus: a pair of passive periphrastics; in footnote 69, Kepler explains that the torso being accelerated by the explosion would collide with any limbs that lie on top of the body

nē . . . gestētur: negative purpose clause, often introduced by "lest"

ut . . . dīvidātur: purpose clause

[70]Tunc excipit nova difficultās, ingēns frīgus, & [71]prohibita respīrātiō, [72]quōrum illī, ingenitā nōbīs vī, & [73]huic vērō, spongiīs hūmectīs ad nārēs admōtīs, obviam īmus. [74]Cōnfectā prīmā parte itineris, facilior redditur vectiō. [75]Tunc līberō āerī expōnimus corpora, manūsque subtrahimus.

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excipiō, -ere, -cēpī, -ceptus: (here) befall, follow
difficultās, -ātis f.: difficulty
ingēns, -entis: immense, vast
frīgus, -oris n.: coldness
prohibeō, -ēre, -uī, -itus: hold back, restrain
respīrātiō, -ōnis f.: respiration
ingenitus, -a, -um: innate
vīs, vīs f.: (here) power, faculty; (in physics) force
spongia, -ae f.: sponge (from the Greek σπογγιά)
hūmectus = ūmectus, -a, -um: damp
nāris, -is f.: nostril; (pl.) nose
admoveō, -ēre, -mōvī, -mōtus: bring near, apply
obviam (adv.): in the way, against (see below)
cōnficiō, -ere, -fēcī, -fectus: complete
vectiō, -ōnis f.: (the act of) transporting
līber, -era, -erum: free
āēr, āeris m.: air (from the Greek ἀήρ)
expōnō, -ere, -posuī, -positus (+ acc. and dat.): expose to, release to
subtrahō, -ere, -trāxī, -tractus: withdraw, remove (cf. subtract)

---

quōrum illī . . . huic vērō: *the former of which . . . and the latter*; these pronouns refer back to the two issues Kepler listed earlier and serve as the dative indirect objects of *obviam īmus*

ingenitā nōbīs vī: *by means of a power innate to us*; ablative of means

spongiīs hūmectīs ad nārēs admōtīs: *by means of damp sponges applied to the nose*; ablative of means

obviam īmus: *we fight against*; *obviam* with a verb of motion generally describes an encounter (here, a defensive one)

Cōnfectā prīmā parte itineris: ablative absolute with *prīmā parte* as the noun and *cōnfectā* as the PPP

Atque illa in sēsē conglobantur ut arāneī: [76]quae nōs sōlō ferē nūtū trānsportāmus, [77]adeō ut dēnique mōlēs corporea sponte suā vergat in locum prōpositum. [78]Sed parum nōbīs est ūtilis haec ῥοπὴ, quia nimis tarda: itaque nūtū ut dīxī accelerāmus; & praecēdimus iam corpus, nē dūrissimō impāctū in Lūnam damnī quid patiātur.

---

sēsē: emphatic version of *sē*
conglobō (1): gather into a ball, conglomerate
arāneus, -ī m.: spider (cf. French "araignée")
ferē (adv.): nearly, almost
nūtus, -ūs m.: nod; will, power
adeō (adv.): so far
dēnique (adv.): finally
mōlēs, -is f.: mass
corporeus, -a, -um: corporeal, bodily
sponte suā: in its own accord, by itself
vergō, -ere, versī, –: turn
prōpōnō, -ere, -posuī, -positus: place before; intend
parum (+ gen. of an adj.): not very
ūtilis, -e: useful
nimis (adv.): too
tardus, -a, -um: slow
accelerō (1): accelerate
praecēdō, -ere, -cessī, -cessus: go before, lead the way
impāctus, -ūs m.: impact
damnum, -ī n.: harm
patior, -ī, passus sum: suffer

---

quae: the antecedent is *illa*, which in turn refers to *corpora*

nūtū: in footnote 76, Kepler clarifies that he is referring to inertia, first used in its modern sense (a body's inherent resistance to any changes in its motion) in his own *Epitomē*; Kepler believed that inertia is caused by an active resistive "soul," "power," or *nūtus*, in Latin, that resides in every object (cf. Kepler's notion of an *anima mōtrīx* on page 122)

adeō ut: here used temporally as "to the point (in time) that" or simply "until"; in this phrase, *adeō* anticipates a result clause

ῥοπὴ: a Greek feminine noun (which the feminine *haec* modifies) meaning "downward motion," originally in reference to scales tipping; if they were to rely on inertia alone, the daemons would move at the same speed (i.e. ῥοπή) as their passengers, but they need to find a way to move faster in order to execute the final step of this journey to the Moon

nūtū . . . accelerāmus: the daemons likely possess a more supernatural "power" (*nūtus*) within themselves in addition to "inertia" (also *nūtus*) that allows them to accelerate, rather than keep the same speed

nē . . . patiātur: another negative purpose clause that can be introduced in English using "lest"

damnī quid: *anything of harm*, or, better yet, *any harm*; after *nē*, *quis* is equivalent to *aliquis*; this is a common partitive genitive construction with *aliquid*

[79]Solent hominēs, cum expergīscuntur, querī dē ineffābilī membrōrum omnium lassitūdine; ā quā sērō admodum sē recipiunt, ut ambulent.

---

expergīscor, -ī, experrēctus sum: wake up, awake
queror, -ī, questus sum: complain
ineffābilis, -e: unspeakable (cf. ineffable)
lassitūdō, -inis f.: weariness
sērō (adv.): (here) slowly
admodum (adv.): completely
recipiō, -ere, -cēpī, -ceptus: take back; recover

---

ineffābilī: an ablative modifying *lassitūdine* and bracketing the objective genitive *membrōrum omnium*

In Somnium Astronomicum Notae, successīvē scrīptae inter annōs 1620. 1630.

54. Nōn sita est, sed natat potius, sī ad Īnsulae similitūdinem respicimus. Sed hīc iam ad imāginātiōnem vīsūs loquendum fuit. Nam quī in Lūnā esset, omnīnō is Lūnam stāre locō fīxam exīstimāret.

---

astronomicus, -a, -um: astronomical
nota, -ae f.: (here) note, comment
successīvē (adv.): successively, in stages
natō (1): swim; float
potius (adv.): rather
similitūdō, -inis f.: similarity
respiciō, -ere, -spexī, -spectus (+ *ad*): look back at; regard, consider
imāginātiō, -ōnis f.: visualization (i.e. the creation a mental "image")
vīsus, -ūs m.: sight
loquor, -ī, locūtus sum: speak, express
omnīnō (adv.): absolutely; in every respect
stō, stāre, stetī, status: stand, remain
fīgō, -ere, fīxī, fīxus (+ acc. and abl.): fix in
exīstimō (1): reckon, think

---

inter annōs 1620. 1630.: *between the 1620th (and) 1630th years*; Arabic numerals ending in a period signify ordinals

Sed hīc iam ad imāginātiōnem vīsūs loquendum fuit: *But here (and) now it had to be said, for the visualization of the sight*; *loquendum fuit* is a passive periphrastic in the past tense; *ad* can express the purpose for which something is done, e.g. why Kepler decides to characterize the Moon as a stationary island

Nam quī in Lūnā esset, omnīnō is . . . exīstimāret: *For whoever is on the Moon would reckon in every respect . . .* ; *is* refers to *quī*

57. Hoc intellige physicē; sī corpus, cui sua gravitās, hōrae spaciō per 12 mīllia mīlliārium rapiātur in altum. Adde prīvātiōnem āeris (dē quā Num. 71), quā hominēs perīre necesse est, ut piscēs prīvātiōne aquae. ⇒

---

intelligō = intellegō, -ere, -lēxī, -lēctus: understand
physicē (adv.): physically (i.e. "scientifically"; from the Greek φύσις, "nature")
gravitās, -ātis f.: weight; (in physics) gravity
rapiō, -ere, -uī, -tus: carry off
addō, -ere, -didī, -ditus: add
prīvātiō, -ōnis f.: privation
Num. = numerus, -ī m.: number (referring to Kepler's footnotes)
pereō, -īre, -iī, -itūrus: pass away, perish
necesse est: it is inevitable that

---

intellige: an imperative, like *adde* in the next sentence

sī corpus . . . in altum: due to the present subjunctive *rapiātur*, translate this clause as the protasis of a future-more-vivid conditional; the implicit apodosis is the situation described in the main text of the *Somnium*

cui sua gravitās: *which has its own weight*; assume an *est* to complete the dative of possession construction

hōrae spaciō: *within the space of an hour*; ablative of time within which

mīlliārium: a textual variant of *mīliārium*, "(of) miles," seen at the beginning of the excerpt; this genitive is again governed by a form of *mīlle*

in altum: an idiom for "on high" or "to the sky"

dē quā Num. 71: *which Num. 71 (is) about*; assume an *est*; the antecedent of *quā* (in this clause and the next) is *prīvātiōnem*; note 71 simply reads, "See Num. 57 above"

prīvātiōne aquae: ablative of means

57. *(continued)* Nam inter dogmata, quae praestantissimīs physicīs accepta ferō, hoc etiam est, Āeris superficiem cum ipsīs montium altissimōrum fastīgiīs, aut etiam īnferius, terminārī.

---

dogma, -atis n.: dogma, philosophical tenet (from the Greek δόγμα, something "seeming" to be true)

praestāns, -antis: preeminent, distinguished

physicus, -ī m.: natural philosopher (or, in modern terminology, "physicist"; from the Greek φύσις, "nature")

superficiēs, -ēī f.: surface, upper part

fastīgium, -iī n.: summit

īnferus, -a, -um: low

terminō (1): bound

---

quae praesantissimīs physicīs accepta ferō: *acceptum referre* or *ferre* plus a dative indirect object (here, *praesantissimīs physicīs*) means "to accredit" something to someone

hoc: in later Latin, *hoc* is often used to refer to a subsequent indirect statement and so can be translated as "the following"

61. Est inter apparātūs iocōrum & hoc, ut dum applausum ūnīus audiente alterō captāre cēnsēris, feriās & hunc, & proximum. Laudem tamen habet, ut Germānia corpulentiae & edācitātis, sīc Hispānia ingeniī & iūdiciī, & frūgālitātis. ⇒

---

apparātus, -ūs m.: preparation
iocus, -ī m.: joke
applausus, -ūs m.: applause
captō (1): strive after
cēnseō, -ēre, -uī, -us: (here) think
feriō, -īre, –, –: strike, knock
proximus, -a, -um: nearest; (here) next
laus, laudis f.: praise
ut . . . sīc: as . . . so does
corpulentia, -ae f.: corpulence
edācitās, -ātis f.: gluttony
Hispānia, -ae f.: Spain
ingenium, -iī n.: genius (cf. ingenuity)
iūdicium, -iī n.: judgment (specifically, good judgment)
frūgālitās, -ātis f.: frugality

Est inter apparātūs iocōrum & hoc, ut: *there is a difference between the preparation of jokes and this, namely that*; *hoc* points to the subsequent noun clause

audiente alterō: *while another is listening*; an ablative absolute

Laudem tamen habet: *nevertheless holds praise for*; *habet* should not be translated as "has" in the sense of "receives," but as "holds" in the sense of "reserves"

Germānia corpulentiae & edācitātis, sīc Hispānia ingeniī & iūdiciī, & frūgālitātis: *Germānia* and *Hispānia* are the subjects of *habet*; *corpulentiae* and *edācitātis* in the first clause are objective genitives governed by *laudem*, as are *ingeniī*, *iūdiciī*, and *frūgālitātis* in the next

61. *(continued)* In subtīlibus igitur scientiīs, cuiusmodī est Astronomia (& praesertim haec Lūnāris, cōnstāns positiōne īnsolentī, sī quis ex Lūnā spectāret), sī pariter annītantur Germānus & Hispānus, multum hic praevertet. Et libellum igitur hunc praedīxī dērīdiculō futūrum Germānīs, at in aliquō sēnsū, Hispānīs.

---

subtīlis, -e: subtle, exact
igitur: therefore
scientia, -ae f.: science
praesertim (adv.): especially
positiō, -ōnis f.: position
īnsolēns, -entis: unusual
spectō (1): look
pariter (adv.): equally
annītor, -ī, -nīxus sum: press upon; strive, labor (a strengthening of *nītor, -ī, nīxus sum*)
Germānus, -ī m.: German
multum (adv.): very much, greatly
praevertō, -ere, -ī, -versus: go before; surpass
libellus, -ī m.: little book (diminutive of *liber, librī m.*)
praedīcō, -ere, -dīxī, -dictus: predict; give notice, warn
dērīdiculum, -ī n.: mockery, ridicule
sēnsus, -ūs m.: sense

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cuiusmodī: *of which kind*; a common merging of *cuius* and *modī*

haec Lūnāris: *this (astronomy) of the Moon*; *haec* refers to *Astronomia*

cōnstāns positiōne īnsolentī: *standing still because of the unusual position* (of the observer); *cōnstāns* modifies the similarly nominative *haec*; *positiōne īnsolentī* is an ablative of cause; Kepler is this time more careful regarding the apparent immobility of the Moon

sī quis ex Lūnā spectāret: *if anyone were to watch from the Moon*; in a conditional, *quis* is equivalent to *aliquis*

sī pariter annītantur Germānus & Hispānus, multum hic praevertet: the present subjunctives indicate a future-more-vivid conditional; *hic* here means "the latter"

dērīdiculō futūrum Germānīs, at . . . Hispānīs: *will be an object of ridicule for the Germans and Spaniards*; the dative of purpose *dērīdiculō* and the datives of reference *Germānīs, at . . . Hispānīs* form a double dative construction; assume an *esse* to complete the infinitive *futūrum esse* of the indirect statement

sēnsū: the original text reads *cēnsū* (from *cēnsus, -ūs m.*, "census"), but the two consonants were frequently confused for each other in this time period

62. [. . .] Sed documentum ēvidentissimum cognātiōnis huius est in flūxū & reflūxū maris; dē quō vidē introductiōnem meam in *commentāria dē mōtibus stēllae Mārtis*. Lūna in vertice cōnstitūta Ōceanī Atlanticī, Ōceanī Austrālis dictī, Ōceanī Ēōī, Ōceanī Indicī attrahit aquās terrae globō circumfūsās; ⇒

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documentum, -ī n.: example
ēvidēns, -entis: evident
cognātiō, -ōnis f.: relationship
flūxus, -ūs m.: flow (in this case, of tides)
reflūxus, -ūs m.: a flowing back; (again, of tides) ebb
intrōductiō, -ōnis f. (+ *in* and acc.): introduction to
commentārium, -iī n.: commentary
mōtus, -ūs m.: movement, motion
stēlla, -ae f.: star; planet (cf. the Ancient Greek term πλάνητες ἀστέρες, "wandering stars")
Mārs, Mārtis m.: Mars (of the god and, here, of the planet)
vertex, verticis n.: top, highest point
cōnstituō, -ere, -ī, -ūtus: set, fix
ōceanus, -ī m.: ocean (from the name of the Greek god Ὠκεανός)
Atlanticus, -a, -um: Atlantic (the ocean past the Atlas mountains)
austrālis, -e: southern
ēōus, -a, -um: eastern (from the Greek ἠῷος, -α, -ον, "of the morning")
Indicus, -a, -um: Indian
attrahō, -ere, -trāxī, -tractus: draw towards, attract
globus, -ī m.: globe
circumfundō, -ere, -fūdī, -fūsus (+ dat.): pour around; (here, in the pass.) surround

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dē quō: the antecedents of the relative pronoun are *flūxū & reflūxū*, where the discrepancy in number is typical

vidē: an imperative

Ōceanī Atlanticī . . . Ōceanī Indicī: partitive genitives for *aquās*

62. *(continued)* quā prōlectātiōne sit ut aquae undique properantēs ad locum spaciōsum, terrārumque continentibus nōn interclūsum, quī Lūnae ad perpendiculum est subiectus, dētegant littora. ⇒

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prōlectātiō, -ōnis f.: enticement
undique (adv.): from all sides
properō (1): hurry forward, rush
spaciōsus = spatiōsus, -a, -um: spacious
continēns, -entis m.: continent (a substantive form of the present participle of *contineō, -ēre, -uī, -tentus*, "hold together, bound")
interclūdō, -ere, -clūsī, -clūsus: shut off, hinder; (here, for a space) enclose
ad perpendiculum: perpendicularly (from *perpendiculum, -ī n.*, "plumb-line," a surveying tool composed of a string with a lead weight hanging at the end that keeps the string vertical)
subiciō, -ere, -iēcī, -iectus (+ dat.): throw under, set under
dētegō, -ere, -tēxī, -tēctus: uncover, expose, lay bare
littus = lītus, -oris n.: shore

---

quā prōlectātiōne: *through this enticement*; ablative of means, where the relative pronoun functions as a demonstrative

sit ut: *it happens that*; the noun clause introduced by *ut* serves as the subject of *sit*

ad locum spaciōsum, . . . interclūsum, quī . . . est subiectus: both the participle *interclūsum* and the relative pronoun *quī* refer to the masculine accusative *locum spaciōsum*

62. *(continued)* Interim vērō dum sunt in itinere, Lūnā ā vertice ūnīus Ōceanī abeunte, agmen aquārum impingēns in littus Occidentāle, dēsertumque ā tractrīce causā, refunditur, littoribusque Orientālibus vicissim īnfunditur. In *Harmonicīs* lib. IV. Cap. ult. dēlīberāvī dē causā īnsuper aliā flūxūs & reflūxūs Maris; quae huic quidem connexa est. Sed ea, quam hīc tangō, sufficit īnstitūtō praesentī. [. . .]

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interim (adv.): meanwhile

abeō, -īre, -iī, -itus: go away, leave

agmen, agminis n.: a moving forwards, (of waters) current

impingō, -ere, -pēgī, -pāctus: strike, dash against

occidentālis, -e: western (in reference to the "setting" sun)

dēserō, -ere, -uī, -tus: leave, abandon

tractrīx, -īcis: attractive (the feminine counterpart to *tractor, -ōris m.*, repurposed as a feminine adjective)

refundō, -ere, -fūdī, -fūsus: pour back

orientālis, -e: eastern

vicissim (adv.): on the contrary; (here) in turn

īnfundō, -ere, -fūdī, -fūsus: pour upon

harmonicē, -ēs f.: harmony (from the Greek ἁρμονική ἐπιστήμη, "music theory")

lib. = liber

Cap. = caput, capitis n.: (when referring to a book or a piece of writing) chapter

ult. = ultimus, -a, -um: last, final

dēlīberō (1): deliberate, ponder

īnsuper (adv.): in addition

connectō = cōnectō, -ere, -nexuī, -nexus: connect, relate

tangō, -ere, tetigī, tāctus: (here) touch upon, mention

sufficiō, -ere, -fēcī, -fectus: (here) be sufficient

īnstitūtum, -ī n.: purpose

praesēns, -entis: present, at hand

Lūnā ā vertice ūnīus Ōceanī abeunte: ablative absolute with the noun *Lūnā*

In *Harmonicīs* lib. IV. Cap. ult.: read as *in capite ultimō librī quārtī Harmonicēs*; *Harmonicīs* is likely a printer's error for *Harmonicēs*, the expected genitive singular form of this Greek borrowing

aliā: modifies *causā*; in the *Harmonicē*, Kepler entertains the idea that the tides bear witness to the respiration of the Earth, which has harmonized itself with the motions of the Moon in the same way that the sleep cycles of animals have synchronized with the day-night cycle

quae: the antecedent is the feminine *causā*; *huic* also modifies an implicit *causae*; both explanations of the tides appealed to Kepler as an astrologer — indeed, a long-distance force emanated by celestial bodies could be seen as a mechanism not only for the tides, but also for the planets' supposed effects on human behavior or fortune

66. Gravitātem ego dēfīniō virtūte, magnēticae similī, attractiōnis mūtuae. Huius vērō attractiōnis maior vīs est in corporibus inter sē vīcīnīs quam in remōtīs. Fortius igitur resistunt dīvulsiōnī ūnīus ab alterō, cum adhūc sunt vīcīna invicem.

---

dēfīniō, -īre, -īvī, -ītus: define
virtūs, -ūtis f.: (in physics) force
magnēticus, -a, -um: magnetic (from the Greek Μαγνῆτις λίθος, a stone with magnetic properties historically found in Magnesia)
similis, -e: similar
attractiō, -ōnis f.: attraction
mūtuus, -a, -um: mutual
vīcīnus, -a, -um: near, close
remōtus, -a, -um: distant, remote
fortiter (adv.): strongly
resistō, -ere, -stitī, – (+ dat.): resist
dīvulsiō, -ōnis f.: a tearing apart, separation (cf. divulsion)
invicem (adv.): in turn; (here) mutually

---

virtūte, magnēticae similī: *similī* modifies *virtūte*, an ablative of means; *magnēticae* modifies an implicit *virtūtī*; Kepler compares the attractive force of gravity to a magnetic force, which had been recently investigated by William Gilbert in his 1600 book *Dē Magnēte* — in fact, Gilbert proved that the Earth functioned as a large magnet, and Kepler postulated that all celestial bodies, in particular the sun, share a similar *anima mōtrīx* ("motive soul") that could account for the elliptical orbits of planets. He also assumed that it emanated only along the ecliptic, which would explain the fact that all planets orbit in the same plane (Cohen, 1985)

attractiōnis mūtuae: an appositional genitive governed by *virtūte*

Fortius igitur resistunt dīvulsiōnī ūnīus ab alterō: *Therefore, they resist more strongly the separation of one from another*; the comparative of *fortiter* is *fortius*

74. Nīmīrum ubi corpus ēreptum ex orbe virtūtis magnēticae tellūris tantō spaciō ut iam praeponderet virtūs magnētica orbis Lūnae.

---

nīmīrum (adv.): surely, doubtlessly
ēripiō, -ere, -uī, -reptus: snatch away, pull out
orbis, -is m.: orb, sphere; region
tellūs, -ūris f.: Earth (cf. tellurium)
praeponderō (1): be of more weight; have more influence

ēreptum: assume an *est* to complete the perfect passive verb

tantō spaciō: an ablative of degree of difference

ut iam praeponderet virtūs magnētica orbis Lūnae: result clause, as anticipated by the earlier *tantō*

75. Suspēnsīs virtūtibus magnēticīs Terrae & Lūnae contrāriā tractiōne; perinde est, ac sī nūllum corpus quōquam prōlectāret. Tunc igitur corpus ipsum, ut tōtum, trahit sua membra, ut partēs minōrēs tōtō.

---

suspendō, -ere, -ī, -pēnsus: hang; (here) suspend, interrupt
contrārius, -a, -um: opposite
tractiō, -ōnis f.: pulling
perinde (adv.): in the same manner
quōquam (adv.): anywhere
prōlectō (1): entice
trahō, -ere, trāxī, tractus: pull

Suspēnsīs virtūtibus magnēticīs Terrae & Lūnae: ablative absolute with *virtūtibus magnēticīs* as the noun and *suspēnsīs* as the participle

contrāriā tractiōne: ablative of means

perinde . . . ac sī: *in the same manner as if* or simply *just as if*

sī . . . prōlectāret: again, an imperfect subjunctive verb is required in this present contrary-to-fact protasis

ut partēs minōrēs tōtō: *as parts (that are) smaller than the whole*; this clause parallels the *ut tōtum* earlier in the sentence; *tōtō* is an ablative of comparison

78. Initiō quidem parum; at proximē Lūnam valdē multum, ut statim sequitur. Itemque parum ūtilis (plānē nihil) annītentibus: nītentēs vērō corpus tollere etiam tum iuvat, cum adhūc terra praepollet: Vidē suprā Num. 62.

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initium, -iī n.: beginning
parum est: (without an adjective) it is not sufficient
valdē (adv.): very
item (adv.): likewise
nihil: (as an emphatic *nōn*) not at all
iuvō, -āre, iūvī, iūtus (+ acc. and inf.): help (someone) to
praepolleō, -ēre, -uī, –: surpass in power
suprā (adv.): above

Initiō quidem parum: assume an *est*; *initiō* is an ablative of time when

proximē Lūnam: *very near to the Moon*; *proximē* can take an accusative alone when describing the proximity to an object

valdē multum: *very much*, in contrast to the previous *parum*

parum ūtilis: again, assume an *est*; like earlier, it means "not very useful"

annītentibus: *for the laborers*; this substantive participle is a dative of reference

nītentēs: a substantive like *annītentibus* above (it even has the same meaning); along with *tollere*, it is part of an accusative-infinitive construction set up by *iuvat*

Vidē suprā Num. 62.: *vidē* is an imperative again; Kepler refers back to his note regarding the tides, which serve as proof that the Moon's gravity does have a noticeable effect even at the surface of the Earth, where the planet's own gravity is far stronger

Chapter 5 — Leibniz: *Dē Arte Combīnātōriā*

The 20th-century physicist and founder of statistical thermodynamics Ludwig Boltzmann once referred to Gottfried Wilhelm Leibniz (1646 CE – 1716 CE) as "the last man who still could unite the entire knowledge of his time in one single head."[1] While this remark was meant to highlight the growing interdependence of scientific fields and the impossibility of any one scholar mastering them all, it is also a testament to both the extraordinary breadth of Leibniz's knowledge and his ability to draw connections across his areas of interest.

Most famously, Leibniz shares with Newton the credit for developing the calculus of differentiation and integration: Not only did Leibniz's notation of d (from *differentia*) and $\int$ (from *summa*) become preferable to Newton's dot notation ($\dot{x}$), but his formulation of calculus in terms of infinitesimals has since been formalized through generalizations[2] of real numbers (Probst, 2013). The idea of infinitesimals echoes his metaphysics, which revolved around the concept of monads, a thing so fundamental that it lacks even spatial extent (which, according to Leibniz, implies that matter itself is an illusion) but possesses action and perception to account for macroscopic changes and the existence of a soul.

The monadology is in turn dependent on Leibniz's five great principles, which he believed could form the basis for all logic (Look, 2020). An example of such a principle is the Principle of Continu-

[1] This translation is taken from Boltzmann (1974).

[2] In the so-called hyperreals, one can define hyperintegers ω greater than any natural number and infinitesimals $\epsilon = \frac{1}{\omega}$ less than any unit fraction. With some more groundwork, derivatives and integrals can be directly expressed in terms of infinitesimals (corresponding to limits) and hyperintegers (corresponding to the number of terms in an integral sum).

ity, commonly summarized in Latin as *nātūra nōn facit saltūs*, "nature does not make jumps," which has its origins in mathematics but would become most prominent in the Darwinian theory of evolution (Perrot, 1977). Indeed, Leibniz even published an early work on geological history, the *Prōtogaea*, that correctly identified fossils as remnants of past life, another insight in Darwin's work. Of course, Leibniz was unfortunately held back by the prevailing assumption at the time that the world was created within a relatively short time frame, especially when compared to the billions of years accepted nowadays (Fortey, 2008). In his early legal education at the University of Leipzig, he even tried to apply mathematical principles to the law (Artosi et al., 2013).

A running thread across many of these varied interests was Leibniz's goal to determine, following the Euclidean tradition, the simplest components or axioms of any physical or intellectual matter. The seeds of this project were present and already sprouting quite prominently in Leibniz's earliest major work, and the one from which the excerpts in this reader are taken, the *Dissertātiō Dē Arte Combīnātōriā* (Leibniz, 1666),[3] published when he was only twenty years old. While the book is mainly devoted to combinatorial mathematics, it covers a wide range of surprisingly related topics, including the nature of colors and the number of possible rearrangements of words in a verse that fit a given poetic meter.

[3]Full Latin subtitle: *In quā ex Arithmēticae fundāmentīs Complicātiōnum ac Trānspositiōnum Doctrīna novīs praeceptīs exstruitur, & ūsus ambārum per ūniversum scientiārum orbem ostenditur; nova etiam Artis Meditandī,* seu *Logicae Inventiōnis sēmina sparguntur. Praefīxa est Synopsis Tractātūs, & additāmentī locō Dēmōnstrātiō Existentiae Deī, ad Mathēmaticam certitūdinem exācta.* ("In which a doctrine of Combinations and Permutations is constructed from the foundations of Arithmetic through new principles, and the utility of both is shown through the universal domain of the sciences; the new seeds for an Art of Thinking, or a Logic of Invention, are also sown. An outline of the treatise is prefixed, as well as, in the place of an appendix, a Demonstration of the Existence of God, exact to a Mathematical certitude.")

The first excerpt below specifically introduces the fundamentals for his *charactēristica ūniversālis* or "universal notation" readily intelligible to every reader. The construction of this universal language was a lifelong mission of Leibniz, inspired by the *Ars Magna* of the 13th-century philosopher and theologian Ramon Llull, which was a work on a universal system of logic (Gray, 2018). Leibniz argued that, in order to be of use for both comprehension (as a practical language) and future invention (as the keystone for the toolbox of science), the *charactēristica ūniversālis* must encode the building blocks of philosophical concepts in a straightforward and likely pictographic way, akin to Chinese or Egyptian scripts. In fact, as an early linguist, Leibniz argued that all modern languages are descended from a sort of *charactēristica ūniversālis* but have evolved to the point that the connections between ideas and the symbols representing them were severed (Hawkins, 2018). Accordingly, he believed that remnants of this universal language still exist or, at the very least, that there have been attempts at reviving it: For instance, Leibniz interpreted the tones of the Chinese language as an auditory realization of binary numbers, possibly suggesting it was originally created as a *charactēristica ūniversālis* (Hawkins, 2018), and he hypothesized a relationship between certain Chinese divination symbols and his own base-2 number system (Sypniewski, 2005).

His brief foray into binary arithmetic and, more importantly, his desire to construct a machine that could produce and verify all pieces of knowledge likely qualify Leibniz as an ancestor of computer science. In a 1685 letter to Philipp Spener, he wrote, "The only way to rectify our reasonings is to make them as tangible as those of the Mathematicians, so that we can find our error at a glance, and when there are disputes among persons, we can simply say: Let us cal-

culate, without further ado, to see who is right."[4] Although Leibniz never realized this project, later characterizing it as immature and overly ambitious, he did construct a prototype for an analog calculator incorporating multiplication and division, a modest first step towards modern computers (see Figure 1). Leibniz's enthusiasm for the fantastic capabilities of his hypothetical machine naturally prompted skepticism. Notably, Jonathan Swift's 1726 *Gulliver's Travels* includes a satirical depiction of such a contraption (see Figure 2) that would allow "the most ignorant person, at a reasonable charge, and with a little bodily labour," to "write books in philosophy, poetry, politics, laws, mathematics, and theology, without the least assistance from genius or study" (Gray, 2018). Although this description was meant, at the time, to sound implausible, in retrospect it appears eerily prescient of the recent spread of artificial intelligence.

In the second excerpt below, Leibniz explains how to calculate the number of permutations directly by means of factorials, which he introduces using a humorous word problem concerning the seating of guests at a table. The three problems that precede it involve the number of ways to choose k objects out of n, quantities known as binomial coefficients, so called due to their presence in the expansion of a binomial $(x+y)^n$. Instead of the modern notation $\binom{n}{k}$, Leibniz employs the slightly awkward terminology of *com2nātiō*, *com3nātiō*, etc. as an etymological generalization of the Latin term *com-bī-nātiō*. He initially proposes the recursive formula $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ (unaware of the independent discovery of this combinatorial identity by Blaise Pascal) before taking up most of the book to derive the closed-form expression $\binom{n}{k} = \frac{n!}{k!(n-k)!}$. The mathematical content of the

[4]This quote and the next are cited in Gray (2018).

Dē Arte Combīnātōriā may seem somewhat lackluster in comparison to his contributions to the theory of information. Leibniz himself seems to have been sufficiently embarrassed by an unofficial republication of the dissertation in 1690 to have felt the need to openly acknowledge his mathematical inexperience when he wrote the *Dē Arte Combīnātōriā* (Mugnai et al., 2020). However, Leibniz's combinatorial studies turned out to have more far-reaching applications than even he could have anticipated, such as the Gaussian distribution, now ubiquitous in statistics, and the entropy calculations of Boltzmann, central to the development of statistical mechanics and thermodynamics.

Source Texts: Our Latin text comes from Leibniz (1666), with a single emendation from Gerhardt (1875):

Page 137	*(& numerentur; fīant deinde Lexica)*	*& numerentur; fīant deinde Lexica, ubi vōcēs*

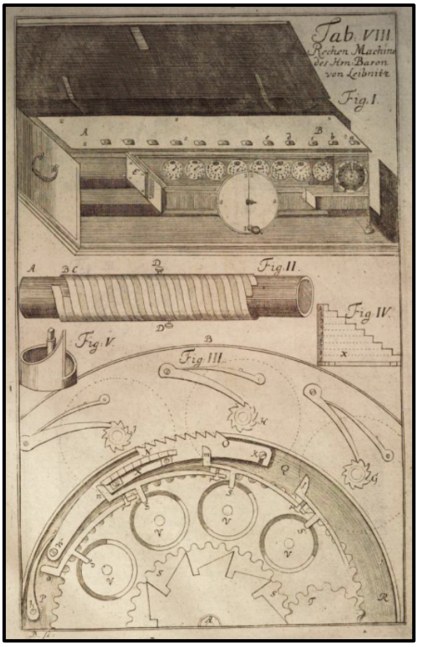


Figure 1: Leibniz's calculator, dubbed the *īnstrūmentum arithmēticum*. Image credit: *Theatrum Arithmetico-Geometricum, das ist: Schau-Platz der Rechen und Mess-Kunst*, public domain.

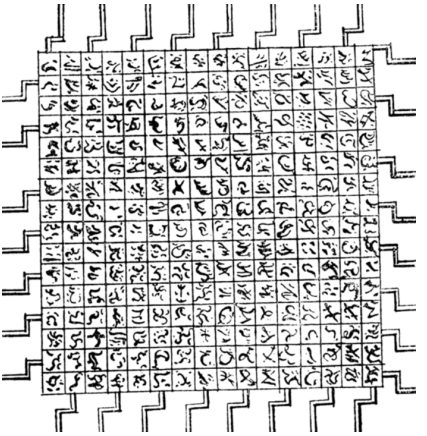


Figure 2: A depiction of The Engine in the floating city of Lagado from *Gulliver's Travels*. Image credit: Wikimedia Commons, public domain.

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Dissertātiō Dē Arte Combīnātōriā Ūsūs Probl. I. & II.

[. . .] Ex hīs quae dē Arte complicātōriā Scientiārum, seu Logicā inventīvā disseruimus, cuius quasi praedicāmenta eiusmodī Terminōrum tabulā absolverentur, ⇒

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dissertātiō, -ōnis f.: dissertation

ars, artis f.: art

combīnātōrius, -a, -um: combinatorial

ūsus, -ūs m.: use, practice; application

Probl. = problēma, -atis n.: problem (from the Greek πρόβλημα, something "put forward")

complicātōrius, -a, -um: binding; combinatorial (from *complicō (1)*, "to fold")

scientia, -ae, f.: science (not limited to natural philosophy)

seu: or

logica, -ae f.: logic (from the Greek λογική, the study of "reason")

inventīvus, -a, -um: inventive

disserō, -ere, -uī, -tus: (here) examine, discuss

quasi: as if; like; almost

praedicāmentum, -ī n.: category, predication

terminus, -ī: boundary, endpoint; (in math) term, value

tabula, -ae f.: table (i.e. a data table)

absolvō, -ere, -ī, -solūtus: release; (here) complete, resolve

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Ex hīs quae dē . . . disseruimus: *from these things which we discussed concerning . . .* or, more idiomatically, *from our discussions on . . .* ; *hīs* is an ablative neuter plural and the antecedent of the neuter accusative plural *quae*; take *ex hīs* closely with *fluit* on the next page

dē Arte complicātōriā Scientiārum, seu Logicā inventīvā: Leibniz is referring to two applications of his combinatorial art from earlier in this section: in the sciences, where the four elements of antiquity can combine only in compatible pairs (e.g. earth and water, but not water and fire); and in the "inventive Logic" or the study of arguments, where the number of different ways to create a predicate statement is sought (i.e. a sentence of the order "subject — verb — predicate")

Arte complicātōriā: Leibniz's preferred term for combinatorics, comprising the mathematics of combinations (the *ars combīnātōria* referenced in the work's title) as well as its applications

praedicāmenta: Leibniz references Aristotle's ten categories or κατηγορίαι for the descriptions of a subject: substance, quantity, quality, conceptual relation, place, time, relative position, condition, action, and affect

eiusmodī: *of such a kind* or simply *such*; a common merging of *eius* and *modī* that goes with *tabulā*

absolverentur: *would be resolved*; a subjunctive in a relative clause of characteristic

(continued) fluit velut Porisma, seu ūsus XI., "Scrīptūra Ūniversālis," id est cuicunque legentī, cuiuscunque linguae perītō, intelligibilis, quālem hodiē complūrēs virī ērudītī tentārunt, [. . .]

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fluō, -ere, flūxī, flūxus: flow, arise, come forth

velut: even as, like

porisma, -atis n.: corollary (from the Greek πόρισμα, something immediately "gained" from a previous result)

scrīptūra, -ae f.: writing, script

ūniversālis, -e: universal

id est: that is (cf. the abbreviation i.e.)

quīcumque, quaecumque, quodcumque (-cunque): whoever, whatever; every, each

legēns, -entis m.: (substantive PAP of *legō, -ere, lēgī, lēctus*) reader

perītus, -a, -um (+ gen.): experienced in, skillful in

intelligibilis = intellegibilis, -e: understandable, intelligible

quālis, -e: of which kind, the sort of which

hodiē (adv.): today, in the present

complūrēs, -a: several

ērudītus, -a, -um: learned, educated

tentō = temptō (1): (here) try, attempt

---

fluit velut Porisma: *arises like a corollary*; the subject of *fluit* is the subsequent *Scrīptūra Ūniversālis*

seu ūsus XI.: *or (as) the 11th application* of Leibniz's combinatorics; Roman or Arabic numerals ending in a period signify ordinals

id est cuicunque legentī, cuiuscunque linguae perītō, intelligibilis: *intelligibilis* is in apposition to *Scrīptūra Ūniversālis*; *perītō*, which modifies the dative of reference *cuicunque legentī*, governs the genitive *cuiuscunque linguae*

quālem hodiē complūrēs virī ērudītī tentārunt: *quālem* introduces the adjectival clause; *tentārunt* is a syncopation of *tentāvērunt*

Is requīrit ut cōnstruātur Lexicon Latīnum, tanquam fundāmentum, & in eō dispōnantur vōcēs ōrdine pūrē alphabēticō & numerentur; fiant deinde Lexica, ubi vōcēs in singulīs linguīs dispositae nōn alphabēticē, sed quō ōrdine Latīnae dispositae sunt ipsīs respondentēs.

---

requīrō, -ere, -quīsīvī, -quīsītus: seek; ask, demand; require
cōnstruō, -ere, -strūxī, -strūctus: construct
lexicon, -ī n.: lexicon (from the Greek λεξικόν βιβλίον)
latīnus, -a, -um: Latin
tanquam = tamquam (adv.): as much as; like, as
fundāmentum, -ī n.: foundation, basis
dispōnō, -ere, -posuī, -positus: set in order, arrange
vōx, vōcis f.: (here) word
ōrdō, -inis m.: order
pūrē (adv.): purely; clearly; simply
alphabēticus, -a, -um: alphabetical
numerō (1): number
singulī, -ae, -a: individual
alphabēticē: alphabetically (adverbial form of *alphabēticus, -a, -um* above)
ipse, -a, -um: (here, simply as a pronoun) he, she, it
respondeō, -ēre, -ī, -spōnsus (+ dat.): (here) correspond to

Is requīrit ut cōnstruātur Lexicon Latīnum: *ut* introduces a set of indirect commands (a.k.a. jussive noun clauses, each containing a subjunctive verb) functioning as the object of *requīrit*; *is* refers to the German alchemist and polymath Johann Joachim Becher, who, in his 1661 work *Charactēr, prō nōtitiā linguārum ūniversālī* ("A style for the universal notation of languages"), proposed the numeric universal language system that Leibniz here describes and then criticizes — Becher even includes a numbered list of 10,283 Latin words, from *ab* to *Zürich*!

ōrdine . . . alphabēticō: *in alphabetical order*; the ablative *ōrdine* simply means "in an order"

Lexica: the plural form of *Lexicon*

ubi vōcēs . . . dispositae nōn alphabēticē: the antecedent of *ubi* is *Lexica*; understand an implied *sunt* to complete the perfect passive indicative verb *dispositae sunt*

quō ōrdine: *in the order in which*; introduces a relative clause with the verb *dispositae sunt*

Latīnae dispositae sunt ipsīs respondentēs: *respondentēs* modifies *Latīnae*, which in turn modifies an implicit *vōcēs*; *ipsīs* refers to the words of the vernacular languages

Scrībantur igitur quae ab omnibus intelligī dēbent numerīs, & quī legere vult, is ēvolvat in Lexicō suō vernāculō vōcem datō numerō signātam, & ita interpretābitur.

---

igitur: therefore

intelligō = intellegō, -ere, -lēxī, -lēctus: understand

numerus, -ī m.: number

ēvolvō, -ere, -ī, -volūtus: unroll; read (i.e. roll open a scroll), peruse

vernāculus, -a, -um: (in linguistics, referring to a native language) vernacular; non-Latin

datus, -a, um: given

signō (1): mark, designate, express

interpretor, -ārī, -ātus sum: explain, interpret; understand

Scrībantur igitur quae ab omnibus intelligī dēbent numerīs: *scrībantur* is a jussive subjunctive ("let . . . be written"), with its subject being the implicit antecedent *ea* of the relative clause introduced by *quae*; *intelligī* is a present passive infinitive with an ablative of agent *ab omnibus*; lastly, *numerīs* is an ablative of means in the main clause

quī legere vult, is ēvolvat: *whoever wants to read (a word) should peruse*; the pronoun *is* refers to *quī*; again, *ēvolvat* is a hortatory subjunctive

signātam: a PPP modifying *vōcem* with an ablative of means *datō numerō*

Ita satis erit legentem vernāculam intelligere & eius Lexicon ēvolvere; scrībentem necesse est (nisi habeat ūnum adhūc Lexicon suae linguae alphabēticum ad numerōs sē referēns) & vernāculam & latīnam tenēre, & utrīusque Lexicon ēvolvere.

---

satis est: it is sufficient
scrībēns, -entis m.: writer
necesse est: it is necessary that
adhūc (adv.): (here) in addition
referō, -ferre, rettulī, relātus: (here) refer
teneō, -ēre, -uī, -tus: hold, keep; know
uterque, utraque, utrumque: both

---

satis erit legentem vernāculam intelligere: *it will be sufficient that the reader understand the vernacular (language)*; *satis erit* requires the complementary infinitive *intelligere*, whose accusative subject is *legentem* and whose object is *vernāculam* (understand *linguam*)

eius: refers to *vernāculam*

ēvolvere another complementary infinitive of *satis erit*

nisi habeat: *unless he should have*; a future-less-vivid conditional with a present subjunctive verb, whose subject is the writer (referred to earlier by *scrībentem*)

sē referēns: modifies the neuter accusative *Lexicon . . . alphabēticum*, while *sē* refers to *scrībentem*

& vernāculam & latīnam: both adjectives modify an implicit *linguam*

tenēre . . . ēvolvere infinitives dependent on *necesse est*, with *scrībentem* as their mutual subject

Vērum & Hispānī illīus & Becherī artificium & obvium & imprācticābile est: ob synōnyma, ob vōcum ambiguitātem, ob ēvolvendī perpetuum taedium (quia numerōs nēmō unquam memoriae mandābit), ob ἑτερογένειαν phrasium in linguīs.

---

vērum: but, however
Hispānus, -ī m.: Spaniard
Becherus, -ī m.: Johann Joachim Becher (see below)
artificium, -iī n.: profession, art; (here) system
obvius, -a, -um: (here) obvious
imprācticābilis, -e: impractical
ob (prep. + acc.): on account of, because of
synōnymum, -ī n.: synonym (from the Greek συνώνυμον, a word sharing a "name with" another)
ambiguitās, -ātis f.: ambiguity
perpetuus, -a, -um: perpetual, constant
taedium, -iī n.: tedium
nēmō, -inis m./f.: no one, nobody
unquam = umquam (adv.): ever
memoria, -ae f.: memory
mandō (1): entrust; (here, with the dative *memoriae*) commit to memory
phrasis, -is f.: phrase, expression (from the Greek φράσις)

& Hispānī illīus & Becherī: *Hispānī illīus* alludes to a Spaniard (whom Leibniz mentions earlier) who devised a universal language where words are categorized into classes, which are each denoted by a number, and where the grammatical cases of words are denoted by additional characters; all Leibniz says about the identity of the Spaniard is that he was in Rome in 1653, but he is suspected to be Pedro Bermudo (Mugnai et al., 2020); *Becherus* is simply the Latin version of Becher

Ob synōnyma . . . ob ἑτερογένειαν phrasium in linguīs: the repetition of *ob* is an example of polysyndeton; Leibniz's problems with Becher's universal language are likely very familiar to any Latin student: two words in a language may have a slight difference in meaning but are both translated as the same word in Latin (and hence would be represented by the same number), a word may have multiple meanings (making its corresponding number ambiguous), the consultation of the glossary may eventually become very tedious, and an idiom may be not translatable and hence incomprehensible in another language like Latin

ἑτερογένειαν: accusative singular of the Greek word ἑτερογένεια, meaning "heterogeneity"; note Leibniz's use of Greek technical terms, which was common ever since the Roman Republic and especially in philosophical literature

Vērum cōnstitūtīs Tabulīs vel praedicāmentīs artis nostrae complicātōriae maiōra ēmergent. Nam Terminī prīmī, ex quōrum complexū omnēs aliī cōnstituuntur, signentur notīs; hae notae erunt quasi alphabētum.

---

cōnstituō, -ere, -ī, -ūtus: set up, erected, constructed
ēmergō, -ere, -mersī, -mersus: come forth, emerge
prīmus, -a, -um: (here) primary, fundamental
complexus, -ūs m.: binding; (in Leibniz's usage) combination
nota, -ae f.: sign, character
alphabētum, -ī n.: alphabet

---

cōnstitūtīs Tabulīs vel praedicāmentīs artis nostrae complicātōriae: *after the tables and predications of our combinatorial art have been constructed*; ablative absolute with the PPP *cōnstitūtīs*

maiōra: as a substantive, "greater things"

ex quōrum complexū: *from the combination of which*

signentur: *should be written*; a potential subjunctive

Commodum autem erit notās quam maximē fierī nātūrālēs, v.g. prō ūnō pūnctum, prō numerīs pūncta; prō relātiōnibus Entis ad Ēns līneās, prō variātiōne angulōrum aut terminōrum in līneīs genera relātiōnum.

---

commodum est: it is agreeable, it is convenient
nātūrālis, -e: natural (i.e., here, with no artificial barriers to learning, such as necessary lexicons)
v.g.: for instance (from *verbī grātiā*, "for the sake of example")
pūnctum, -ī n.: point
relātiō, -ōnis f.: (here) relation
ēns, entis n.: thing
līnea, -ae f.: line
variātiō, -ōnis f.: variation (used later on in a mathematical context as "factorial," because the factorial $n!$ counts the number of "variations" or permutations of n distinct objects)
angulus, -ī m.: angle

Commodum autem erit notās quam maximē fierī nātūrālēs: *commodum erit* introduces the indirect statement with subject *notās* and present passive infinitive *fierī*; *quam* with a superlative adjective or adverb means "as . . . as possible," so *quam maximē* can be translated as "as much as possible"

prō relātiōnibus Entis ad Ēns: *for the relations of a thing to a thing*; although in use here and there since Caesar, *ēns* played an important role in the abstract philosophical language of the Scholastics as an artificial present participle for *esse* and a literal translation of the Greek participle ὤν, "being"

Ea sī rēctē cōnstitūta fuerint & ingeniōsē, scrīptūra haec ūniversālis aequē erit facilis quam commūnis, et quae possit sine omnī lexicō legī, simulque imbibētur omnium rērum fundāmentālis cognitiō.

---

rēctē (adv.): correctly
ingeniōsē (adv.): cleverly (cf. ingeniously)
aequē . . . quam: as . . . as
commūnis, -e: common
simul (adv.): at once
imbibō, -ere, -ī, -: (here) absorb
fundāmentālis, -e: fundamental
cognitiō, -ōnis f.: knowledge

Ea sī rēctē cōnstitūta fuerint & ingeniōsē: a connective to a previous clause or sentence (in this case, the neuter plural pronoun *ea*) can sometimes precede *sī* even if it is a part of the protasis; *cōnstitūta fuerint* is a common alternative form of the future perfect passive indicative *cōnstitūta erunt* (which, in a conditional, is frequently translated in English using the present tense)

scrīptūra haec ūniversālis aequē erit facilis quam commūnis: *this universal script will be as easy as (it will be) common*; *commūnis* is a predicate adjective modifying *scrīptūra* (implying that this hypothetical language is so practical that it is sure to be adopted immediately)

quae: the relative pronoun has *scrīptūra* as its antecedent; this relative clause is causal, as indicated by the subjunctive *possit*

Fīet igitur omnis tālis scrīptūra quasi figūrīs geōmetricīs & velut pictūrīs, utī ōlim Aegyptiī hodiē Sīnēnsēs; vērum eōrum pictūrae nōn redūcuntur ad certum Alphabētum seu līterās, quō fit ut incrēdibilī memoriae afflīctiōne opus sit, quod hīc contrā est. Hic igitur est Ūsus XI. complexiōnum, in cōnstituendā nempe polygraphiā ūniversālī. [. . .]

---

tālis, -e: such
figūra, -ae f.: figure
geōmetricus, -a, -um: geometric (from the Greek γεωμετρία, "earth-measuring")
pictūra, -ae f.: picture
utī = ut
ōlim (adv.): once, some time ago, in the past
Aegyptius, -iī m.: Egyptian
Sīnēnsis, -is m.: Chinese
redūcō, -ere, -dūxī, -ductus: (here) reduce
certus, -a, -um: fixed, established
līter a = littera, -ae f.: letter
quō: for which reason
incrēdibilis, -e: incredible, unbelievable
afflīctiō, -ōnis f.: affliction, suffering
opus est (+ abl.): there is a need for
contrā (prep. + acc.): (here, used predicatively) the opposite of
complexiō, -ōnis f.: combination
nempe (adv.): (here) namely
polygraphia, -ae f.: script (the contemporary term for a cryptographic system; from the Greek πολυγραφία, lit. "much writing")

ōlim Aegyptiī hodiē Sīnēnsēs: assume an *et* in between for the sake of clarity, as well as some verb like "had" or "have done"

eōrum: refers to *Aegyptiī* and *Sīnēnsēs*; here, Leibniz is complaining that Egyptian hieroglyphs and Chinese characters do not employ an alphabet, meaning that every word must be remembered separately, but his claim is not entirely true: hieroglyphics have many phonetic glyphs (as later discovered in the 19th century with the help of the famous Rosetta stone), while many Chinese characters are compound words (which may, fittingly enough, be considered a demonstration of Leibniz's combinatorial art!)

fit ut: an idiom for "it happens that," where *ut* introduces a noun clause that functions as the subject of *fit*

quod hīc contrā est: *the opposite of which is the case here*

in cōnstituendā nempe polygraphiā ūniversālī: *namely in constructing a universal language*; a gerundive phrase that is equivalent to a gerund followed by a direct object, namely *in polygraphiam ūniversālem cōnstituendā*

Probl. IV.
Datō Numerō Rērum Variātiōnēs Ōrdinis Invenīre

Solūtiō: "Pōnantur omnēs numerī ab ūnitāte usque ad Numerum rērum, inclūsīvē, in seriē nātūrālī: factus ex omnibus continuē erit quaesītum." Ut: estō Tabulā ה quam ad 24 usque continuāvimus. [. . .]

---

inveniō, -īre, -vēnī, -ventus: find
solūtiō, -ōnis f.: solution (of a problem)
ūnitās, -ātis f.: unity, one
usque (adv.): all the way
inclūsīvē (adv.): inclusive
seriēs, -ēī f.: (in math) series, sequence
factus, -ūs m.: result, product (from *faciō, -ere, fēcī, factus*)
continuē (adv.): continuously
quaesītum, -ī n.: answer (lit. "the thing sought," from *quaerō, -ere, -sīvī, -sītus*)
sum, esse, fuī, futūrus: (here, with abl.) be found in
continuō (1): do one after another; continue, extend

Datō Numerō Rērum: *with the number of things having been given*; ablative absolute

Invenīre: *to find*; infinitive in a problem statement, grammatically in apposition to the *Probl. IV.* opening the chapter title

Pōnantur omnēs numerī ab ūnitāte usque ad Numerum rērum: *Let all numbers be placed from one all the way to the number of things*

in seriē nātūrālī: *in a natural sequence*; Leibniz tells the reader to order the numbers from 1 to n in the order of the natural numbers, from least to greatest

factus ex omnibus continuē: *the product of all (numbers) continuously* (i.e. of the numbers listed above, in order); see below for an explanation of the term *factus*

estō Tabulā ה: *it shall be found in Table* ה; *estō* is a third person singular, future active imperative form of *esse*, commonly employed in legal decrees

ad 24 usque: equivalent to *usque ad 24*, "all the way to 24"

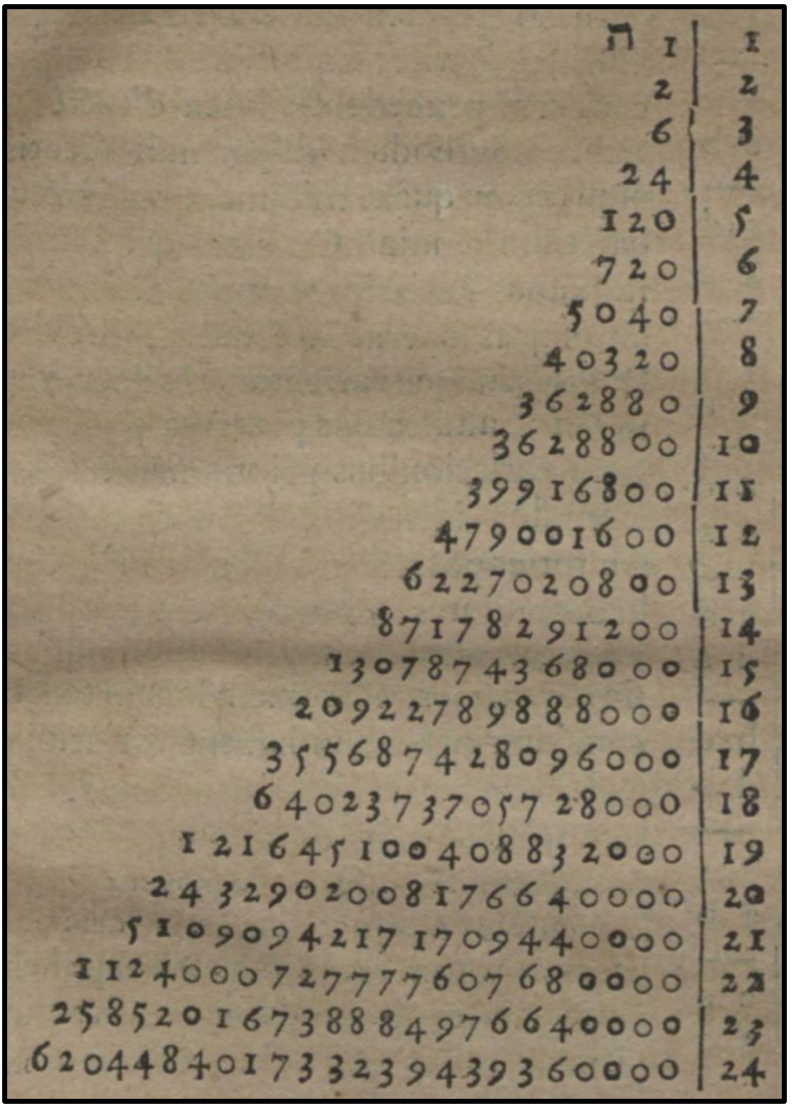


Table ה: Leibniz's table of factorials. The tables in his work are labeled with letters of the Hebrew alphabet. The correct value for 15! is 1307674368000. Image credit: *De Arte Combinatoria*, public domain.

Quemadmodum expōnēns in latere dextrō est ratiō variātiōnis datae ad antecēdentem. Ratiō solūtiōnis erit manifesta, sī dēmōnstrāverimus *expōnentis datī variātiōnem esse factum ex ductū ipsīus in variātiōnem expōnentis antecēdentis*, quod est fundāmentum Tabulae ה.

---

quemadmodum: (here) for instance
expōnēns, -entis m.: exponent
latus, lateris n.: side
dexter, -tra, -trum: right
ratiō, -ōnis f.: reason; (in math) ratio
antecēdēns, -entis: preceding
manifestus, -a, -um: evident, clear (cf. manifest)
dēmōnstrō (1): show, prove, demonstrate
ductus, -ūs m.: (here, the action of) drawing

---

expōnēns: Leibniz's term for the argument of a factorial, derived from the fact that $n!$ is the nth term in a sequence of factorials, in the same way a^n is the nth term in a geometric sequence

ratiō variātiōnis datae ad antecēdentem: *the ratio of the given factorial to the preceding (one)*; in other words, the ratio of $n! = n \cdot (n-1)!$ over $(n-1)!$, which is precisely the exponent n

Ratiō solūtiōnis: *the reason for the solution*; *solūtiōnis* is an objective genitive

sī dēmōnstrāverimus: the future perfect active indicative *dēmōnstrāverimus* is usually translated in English using the present tense when in a conditional; the verb also introduces an (italicized) indirect statement with the infinitive *esse*

***factum ex ductū ipsīus in variātiōnem expōnentis antecēdentis*:** *the result from the drawing of itself* — i.e. the given exponent — *upon the factorial of the preceding exponent*; the area of a rectangle is the product of the lengths of its sides, so, when calculating the product of the exponent n and the previous factorial $(n-1)!$, we can imagine drawing a segment of length n perpendicularly to an existing segment of length $(n-1)!$, and the resulting area will be the desired product; *factus ex ductū . . . in* is therefore a natural description of a product, especially at a time when the foundation of all math was Euclidean geometry

Igitur quaerunt quotiēs datae quotcunque persōnae ūnī mēnsae aliō atque aliō ōrdine accumbere possint. Drexelius in *Phaëthonte orbis, seu dē vitiīs linguae* p. 3. c. 1., ubi "dē linguā ōtiōsā," ita fābulam narrat: Paterfamiliās nesciōquis 6 ad coenam hospitēs invītāverat.

---

quotiēs: how often, how many times

quotcunque = quotcumque (indecl.): however many

persōna, -ae f.: person (originally, the "mask" of an actor)

mēnsa, -ae f.: table

accumbō, -ere, -cubuī, -cubitus (+ dat.): lay oneself down on (e.g. on a *trīclīnium*); take one's place at

Drexelius, -iī m.: Jeremias Drexel (see below)

Phaëthōn, Phaëthontis m.: Phaethon (famous for riding his father Helios's chariot and crashing it onto the Earth)

orbis, -is m.: orb, sphere; (here) world

vitium, -iī n.: fault, vice, sin

p. = pars

c. = caput, capitis n.: (when referring to a book or a piece of writing) chapter

ōtiōsus, -a, -um: idle, pertaining to leisure

fābula, -ae f.: story, fable

narrō (1): narrate

paterfamiliās, patrisfamiliās m.: (lit.) father of the family, (here) head of household

nesciōquis, -quae, -quid: some (lit. "I do not know who")

coena = cēna, cēnae f.: dinner

hospes, hospitis m.: guest

invītō (1): invite

---

datae quotcunque persōnae: *however many given people*; subject of *quaerunt* and *possint*

aliō atque aliō ōrdine: *in one order or another* or *in different orders*

possint: subjunctive verb in an indirect question introduced by *quotiēs*

Drexelius in *Phaëthonte orbis, seu dē vitiīs linguae* p. 3. c. 1., ubi "dē linguā ōtiōsā": read the page reference as *parte tertiā, capite prīmō, ubi (dīcit) "dē linguā ōtiōsā"*; Leibniz cites the book *Orbis Phaëthōn: sīve Dē ūniversīs vitiīs Linguae* ("Phaethon of the World: or about the collective sins of Language") written by the Jesuit writer Jeremias Drexel; the work lists a sin corresponding to each Latin letter — except X, Y, and Z, which are reserved for ways to atone for a wicked tongue — and each one is exemplified by a fable, one of which (from the chapter for O, *Dē Ōtiōsā Linguā*) Leibniz retells here

Hōs, cum accumbendī tempus esset, προεδρίαν sibi mūtuō dēferentēs ita increpat: "Quid? An stantēs cibum capiēmus? Īmō nē sīc quidem, quia et stantium necessārius ōrdō est. Nisi dēsinitis, tum vērō ego vōs, nē conquerī possītis, totiēs ad coenam vocābō, quotiēs variārī ōrdō vester potest."

---

sibi: (here) to each other
mūtuō (adv.): alternately, in return, mutually
dēferō, -ferre, -tulī, -lātus (+ dat.): bring, transfer, offer
increpō, -āre, -uī, -itus: (here) rebuke
an: (introducing a question that provides an alternative) or rather
cibus, -ī m.: food
capiō, -ere, cēpī, captus: (in the context of food) take in
īmō = immō (adv.): (with a negative clause) on the contrary, (in this more informal tone) nay
nē . . . quidem: not even
necessārius, -a, -um: necessary
dēsinō, -ere, -siī, -situs: cease, stop
vērō (adv.): in truth; but, and (postpositive)
conqueror, -ī, -questus sum: complain
totiēs . . . quotiēs: as often as
vocō (1): (here) invite
variō (1): vary, change, interchange
vester, -tra, -trum: your (pl.)

Hōs, cum accumbendī tempus esset, . . . ita increpat: *when it was time for taking (their) place, ... he rebukes these men thus*; *hōs* is the object of *increpat*; *accumbendī* is an objective genitive and a gerund; *esset* indicates a circumstantial *cum* clause

προεδρίαν: accusative singular of the Greek word προεδρία, which means "privilege of the front seat" or simply "front seat"

stantēs: *while standing*; a circumstantial use of a participle

et stantium necessārius ōrdō est: *even the order of the people standing is necessary*; hence, the only possible course of action will be to reconvene each day until each order has been entertained exactly once

nē conquerī possītis: negative purpose clause

Hic antequam loquerētur ad calculōs profectō nōn sēderat, ita enim comperisset ad 720 variātiōnēs (tot enim sunt dē 6 expōnente, utī Drexelius illīc 12 pāginīs, & in quālibet pāginā 3 columnīs, & in quālibet columnā 20 variātiōnibus oculāriter mōnstrāvit) totidem coenīs opus esse; quae etsī continuārentur, 720 diēs, id est 10 suprā biennium, absūment.

---

antequam: before
loquor, -ī, locūtus sum: speak
calculus, -ī m.: calculation, count (originally the diminutive of *calx, calcis f.*, "stone," because little stones were used for counting boards)
profectō (adv.): certainly
sedeō, -ēre, sēdī, sessus: sit; (with *ad*) assess
ita: (here) if this were so, then
comperiō, -īre, -ī, -tus: obtain knowledge about, ascertain
ad (prep. + acc.): (here) according to
tot (indecl.): so many
dē (prep. + abl.): (here) on account of, because of
illīc (adv.): in that place (here referring to a location in Drexel's book)
pāgina, -ae f.: page
quīlibet, quaelibet, quodlibet: any, each, every
columna, -ae f.: column
oculāriter (adv.): with the eyes; visually
mōnstrō (1): show
totidem (indecl.): that many
etsī: even if (from *et* + *sī*)
suprā: (here, of time) before
biennium, -iī n.: two years
absūmō, -ere, -psī, -ptus: use up, consume

Hic . . . ad calculōs . . . nōn sēderat: *this man had not assessed the counts*

loquerētur: imperfect subjunctive of the deponent verb *loquor*; the conjunction *antequam* presumably takes a subjunctive here under the influence of the subjunctive verb in the main clause, even if this subordinate clause does not describe a hypothetical

comperisset: *he would have ascertained*; potential pluperfect subjunctive

12 pāginīs, . . . 3 columnīs, . . . 20 variātiōnibus: all ablatives of means; Leibniz mentions Drexel's enumeration of all 720 permutations of "abcdef" across 12 pages, 3 columns, and 20 variations ($12 \cdot 3 \cdot 20 = 720$)

totidem coenīs opus esse: *totidem* refers to the 720 variations or permutations; recall that *opus est* with an ablative means "is needed"

quae etsī continuārentur: treat the relative pronoun as a demonstrative referring back to *coenīs*; as a connective to the previous clause, *quae* can precede *etsī* even if it belongs to the (present contrary-to-fact) protasis

10 suprā biennium: *10 (days) before two years*

Harsdörfferus (*Dēlic. Math.* p. 2. sect. 1. prōp. 32.) hospitēs pōnit 7, ita variātiōnēs, coenae, diēs erunt 5040, id est annī 14, septimānae 10. At Georg. Henischius Medicus Augustānus (*Arithmēticae Perfectae* lib. 7. pāg. 399.) hospitēs vel convīctōrēs pōnit 12, variātiōnēs, coenae, diēs prōdeunt 479001600, ita absūmentur annī 1312333 & diēs 5.

Harsdörfferus, -ī m.: Georg Phillip Harsdörff (see below)

Dēlic. = dēliciae, -ārum f.: pleasures

Math. = mathēmaticus, -a, -um: mathematical (from the Greek μαθηματικός, ultimately from μανθάνειν, "to learn")

sect. = sectiō, -ōnis f.: (here, of a book) section

prōp. = prōpositiō, -ōnis f.: proposition

pōnō, -ere, posuī, positus: (here) put forward, posit

septimāna, -ae f.: week

Georgius Henischius, -iī m.: Georg Henisch (see below)

medicus, -ī m.: physician

Augustānus, -a, -um: pertaining to Augustus (in reference to Henisch's hometown of Augsburg, established by and named after Augustus)

arithmētica, -ae f.: arithmetic (from the Greek ἀριθμός for "number")

perfectus, -a, -um: perfected

lib. = liber

pāg. = pāgina

convīctor, -ōris m.: table companion

prōdeō, -īre, -iī, -itus: come forth

Harsdörfferus (*Dēlic. Math.* p. 2. sect. 1. prōp. 32.): a reference to *Dēlicae Physico-Mathēmaticae. Oder Mathematische und Philosophische Erquickstunden* ("Physico-Mathematical Pleasures. Or Mathematical and Philosophical Refreshment-Hours"), written by the mathematician and poet Daniel Schwenter and then assembled by the jurist and poet Georg Phillip Harsdörff; among its many math puzzles is a similar guest-permutation problem

Georg. Henischius Medicus Augustānus (*Arithmēticae Perfectae* lib. 7. pāg. 399.): Leibniz cites another version of the problem from *Arithmētica Perfecta et Dēmōnstrāta* ("Arithmetic Perfected and Demonstrated"), a math textbook by the physician, educator, and mathematician Georg Henisch

***Arithmēticae Perfectae* lib. 7.:** *the 7th book of* Arithmētica Perfecta

Īmō sī quis in hōc Expōnente tentāre vellet, quod Drexelius in dīmidiō eius effēcit, nempe variātiōnēs oculāriter experīrī, annōs īnsūmeret 110, dēmtō quadrante, et sī singulīs diēbus 12 hōrīs labōrāret & hōrā quālibet 1000 variātiōnēs effingeret. Pretium operae sī Diīs placet!

---

dīmidium, -iī n.: a half
efficiō, -ere, -fēcī, -fectus: work out
experior, -īrī, -tus sum: test, prove; find by experience, discover
īnsūmō, -ere, -psī, -ptus: (here) use up
dēmō, -ere, -psī, -ptus (-tus): take away, subtract
quadrāns, -antis m.: quarter
hōra, -ae f.: hour
labōrō (1): work (cf. labor)
effingō, -ere, -fīnxī, -fictus: express
pretium operae: (lit.) a reward for labor, worthwhile
placeō, -ēre, -uī, -itus (+ dat.): be pleasing, please

---

sī quis in hōc Expōnente tentāre vellet: *quis* in a conditional functions like *aliquis*; the imperfect subjunctive verbs *vellet* (in the protasis), *īnsūmeret*, and *effingeret* (in the apodosis) indicate the contrary-to-fact present conditional

quod Drexelius in dīmidiō eius effēcit: the antecedent of *quod* is the entire previous clause, and *eius* refers to *expōnente* (i.e. *in dīmidiō eius* means "in half of the exponent"); Drexel listed out 6! variations, while Leibniz now considers 12!

experīrī: a complementary infinitive to *vellet*, like *tentāre*

dēmtō quadrante: *with a quarter (year) subtracted*; ablative absolute

et sī: *even if*

singulīs diēbus 12 hōrīs: *for 12 hours within individual days*; while *singulīs diēbus* is a standard ablative of time within which, *12 hōrīs* is an example of the (mostly post-Classical) ablative of extent of time, which is more commonly expressed with an accusative

hōrā quālibet: another common ablative of time within which

Pretium operae: assume an *est* in the main clause

Diīs: irregular dative plural form of *deus*; the plural may be a bit unexpected, but Leibniz is probably referring to the gods of antiquity (indeed, his references to the Christian *DEUS* are normally in all capital letters)

Chapter 6 — Merian: *Metamorphōsis Īnsectōrum Surinamēnsium*

The evocative German name for the tarantula, "Vogelspinne" (literally "bird-spider"), derives from the binomial name *Arānea aviculāria* put forward in 1758 by the taxonomist Carl Linnaeus in light of the spider's unusual prey, hummingbirds. This discovery was first made by Maria Sibylla Merian (1647 CE – 1717 CE), a naturalist, entomologist, engraver, and painter who eventually superseded her father and stepfather in fame even though they were both accomplished artists in their own right. Her skills are evident in her earliest works, the multivolume *Neues Blumenbuch* and *Raupen-Buch*, which document the plants and caterpillars, respectively, native to Germany. After divorcing her husband in 1684, Merian relocated with both her daughters to the Netherlands, where she also joined (and, six years later, left) the Labadists, a sect that had previously counted the famed poet and scholar Anna Maria van Schurman as one of its members. In 1699, intrigued by the increasing influx of exotic specimens from the Americas, Merian set off with her 21-year-old daughter Dorothea to Suriname, a colony under the control of the western counterpart of the infamous Dutch East India Company — remarkably, her voyage was largely self-funded through tutoring and the sale of many of her paintings. Sadly, recurrent bouts of malaria cut the expedition short in September 1701 (Kutschera, 2017). After her return to Europe, Merian began to compile the 1705 *Metamorphōsis Īnsectōrum Surinamēnsium*,[1] with 60 annotated

[1]Full Latin subtitle of the 1719 edition: *In quā, praeter Vermēs & Ērūcās Surinamēnsēs, eārumque admīrandam Metamorphōsin, Plantae, flōrēs & frūctūs, quibus vēscuntur, & in quibus fuērunt inventae, exhibentur. Hīs adjunguntur Būfōnēs, Lacertī, Serpentēs, Arāneae, aliaque admīranda istīus regiōnis animalcula; omnia manū eiusdem Matronae in Amerīcā ad vīvum accūrātē dēpicta, & nunc aerī incīsa. Accēdit Appendix Trānsfōrmātiōnum Piscium in Rānās, & Rānārum in Piscēs.* ("In which, besides the Surinamese Worms and Caterpillars, and their wonderful Metamorphosis, the Plants, flowers, and fruits, which they eat and in which they were found, are shown. Added to these are the Toads, Lizards, Serpents, Spiders, and other wonderful little animals of this region; all

images of Amazonian insects and other assorted wildlife. Although Merian originally wrote the book in Dutch, she must have realized the potential of attracting a greater scholarly audience with a Latin edition, given that she had it translated and published in Latin in the same year (Valiant, 1993). Indeed, this choice was not an uncommon one during the gradual transition from Latin to vernacular languages in early modern science, especially for less widely spoken languages. The Latin excerpts included here are from the 1719 edition, which is identical in most regards to the 1705 edition except for the addition of twelve new pictures, ten of which were actually completed by her daughters (Merian, 1719).

Despite her efforts to balance artistic elements with the prospect for profit (such as through uncolored and hand-painted versions of the images, sold at different prices) (Schrader et al., 2012), the collection was not a commercial success, and her research on Surinamese reptiles and amphibians, which she intended to elaborate on in a sequel, had to be limited to only the few plates at the end of the *Metamorphōsis* (Kutschera, 2017). However, as she had hoped, it had quite a significant impact on the scientific community, establishing her as a reputable entomologist whose visualizations of species were often copied, for instance by Linnaeus in his own *Systēma Nātūrae*. Neverthless, Merian's legacy suffered from mistakes introduced in translations of her work, as well as unskillful copies of her illustrations. Nineteenth-century naturalists, noticing these inaccuracies, suspected that Merian was incorrect in her descriptions of, for instance, the bird-eating spider, even though they never undertook a journey of their own to verify her findings first-hand. Only in the

depicted accurately to life by the hand of the same Matron in America, and now engraved in copper. An Appendix of the Transformations of Fish into Frogs, and Frogs into Fish, is added.")

past few decades has Merian's work enjoyed renewed appreciation (Valiant, 1993).

Merian contributed significantly to the study of the biological development of insect species. She was an opponent of spontaneous generation, a remnant of the Aristotelian worldview that had already become largely obsolete in other fields. In an attempt to explain the lack of observational evidence for the mating and egg laying of some creatures like eels or worms, Aristotle, who has often been characterized as a biologist first and foremost, hypothesized that these animals are so simple that they can arise without parents through the process of spontaneous generation. It was by no means a theory exclusive to Aristotelianism; even Lucretius occasionally endorses spontaneous generation in his *Dē Rērum Nātūrā*, although usually in the context of atoms that recombine randomly, without the involvement of gods, until eventually creating a living being that could reproduce on its own (Lehoux, 2017). During the Middle Ages, spontaneous generation also became entangled with religious interpretations, with some scientists, such as Jan Goedart, connecting the spontaneous generation of insects, specifically of the butterfly (believed to be born from the corpses of caterpillars), to the resurrection of Christ (Etheridge, 2021). The first major challenge to the theory came in the form of an experiment conducted in 1668 by Francesco Redi, who covered glasses containing dead meat with either nothing, or a thin cloth, or a tight seal. Thanks to the observations that maggots appeared only on the veil in the second case and that they were completely absent in the third, Redi deduced that the insects likely did not originate from the glasses' contents, although Redi still insisted that insects could potentially generate from living plant matter, one major oversight of his experiment (Lehoux, 2017). How-

ever, he was able to finally identify the larval form of flies, namely maggots, and even their eggs by observing their growth in captivity (Gottdenker, 1979).

Merian provided similar observational evidence for the elusive eggs of other species which she immortalized in her art, whether in her personal journals (which actually predate Redi's experiment) or in her published compendiums (Valiant, 1993). For instance, on the peacock flower of Figure XLV of the (appropriately titled) *Metamorphōsis*, she depicts the full lifecycle of an Amazonian moth, including the adult, caterpillar, and pupated forms explicitly alluded to in her description of the image (see the first excerpt below), as well as the egg drawn on the rightmost leaf, at a slightly exaggerated scale. By also noting the duration of the caterpillar's metamorphosis, Merian allowed other naturalists to compare it to their own measurements, dispelling the theory of spontaneous generation that, as the name insinuates, was frequently associated with randomness. Elsewhere, Merian displays a crocodile mother protecting its unhatched young, a frog swimming amongst tadpoles (once deemed a separate species) and its gel-covered eggs, and, in Figure XVIII below, a web-crawling spider carrying its young underneath its body. She thereby helped relegate spontaneous generation to the realm of microorganisms, which had been recently discovered using the microscope, until it was replaced by the germ theory of Pasteur in the 19th century.

Furthermore, Merian's work indirectly confronted one of the major issues that emerged alongside the waning influence of Aristotle. Descartes, an early figurehead of the Scientific Revolution, argued that all phenomena, including the vital systems and gen-

eral behavior of animals, can be understood and predicted through the laws of nature. An unquestioned universal application of this mechanistic view would clearly lead to the uncomfortable conclusion that free will would be merely an illusion. Therefore, Descartes advocated for a mind-body dualism, in which the "mind" is a non-physical substance on a separate plane from matter that only humans possess, leaving all other beings, in his words, equivalent to a clock (Kureethadam, 2017). This frame of mind was reflected in the tendency of naturalists at the time to collect specimens from distant lands, then store them in glass cages, and finally illustrate them with each appendage laid out and labeled like the pieces of an engine (Etheridge, 2021). Merian's art pieces are in some ways the converse, pairing her subjects, alive and well, with the host plant from which they suck nectar or take leaves — she would have gained this knowledge naturally as she would have had to raise the insects, often in enclosures, long enough to witness and catalogue their entire lifecycle. In the text for Figure XVIII, she highlights interspecies encounters, most visibly the aforementioned spider preying on a mother hummingbird, as well as the relationships between the animals and local humans, also touched upon in one of the excerpts below. Finally, Merian recognizes the influence of the climate on the behavior and specialization of species, possibly foreshadowing the modern ecological perspective often accredited to Alexander von Humboldt and his *Naturgemälde*: An animal can only be considered within the context of its environment, which includes not merely its predators and prey, but also metrics such as pressure and temperature that are dependent on global features like currents and mountain chains.

Source Text: Our Latin text comes from Merian (1719).

Slavery in the Americas

Figure 1 (facing page): Figure XLV portrays a flower commonly employed by female slaves in Suriname as an abortifacient to prevent their offspring, whose father was often their master, from having to also endure slavery under the policy of *partus sequitur ventrem* ("the offspring follows the womb"). A large part of the agricultural economy in the New World at the time of Merian still consisted of indigenous slaves, who were gradually substituted with Africans imported through the Transatlantic Slave Trade.

In a rare incorporation of the voices of the African slaves, Merian recounts their belief that death within the Americas leads to a joyous resurrection in their home country, a sentiment vividly relayed by future abolitionist poets such as William Bowles or S. T. Coleridge (May, 2007). However, despite describing the cruelties of slavery, Merian never directly promoted abolitionism and in fact relied on slaves who helped her in collecting specimens and informed her of the customs of the local population (they are occasionally credited in the *Metamorphōsis* as "my slaves" or "my Indian").

The plantation owners, meanwhile, were baffled at her interest in plants other than the lucrative sugarcane (Polcha, 2022). The European abolitionist movement would only gain momentum at the end of the 18th century, well after Merian's death, and the Dutch Empire was one of the last to abolish slavery, in 1863. Image credit: *Metamorphosis Insectorum Surinamensium*, public domain.



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Figure 2: In 1990, the German Central Bank dedicated the 500-mark banknote in its BBk III series to Merian, hosting a competition for the creation of a new portrait template. The obverse has Nuremberg, her hometown, in the backdrop, while the reverse depicts a dandelion from her *Raupen-Buch*. Image credit: Wikimedia Commons, work by YavarPS, licensed under CC BY-SA 4.0 (`https://creativecommons.org/licenses/by-sa/4.0/`).

Dissertātiō Dē Generātiōne Et Metamorphōsibus Īnsectōrum Surinamēnsium Explicātiō Figūr. XLV.

Hic *Flōs Pāvōnis*, planta novem pedum altitūdine crēscēns, flōrēs prōdūcit flāvōs & rubicundōs. Sēmen mulieribus exhibētur parturientibus, ad prōmovendōs partūs labōrēs:

---

dissertātiō, -ōnis f.: dissertation
generātiō, -ōnis f.: generation, reproduction
metamorphōsis, -is metamorphosis (from the Greek μεταμόρφωσις)
īnsectum, -ī n.: insect (lit. "a segmented thing")
Surinamēnsis, -e: Surinamese
explicātiō, -ōnis f.: explanation
figūra, -ae f.: figure
flōs, flōris m.: flower
pāvō, -ōnis m.: peacock
planta, -ae f.: sprout, shoot
novem (indecl.): nine
pēs, pedis m.: foot (here, the unit)
altitūdō, -inis f.: height, altitude; depth
crēscō, -ere, crēvī, crētus: grow
prōdūcō, -ere, -dūxī, -ductus: (here) bring forth, produce
flāvus, -a, -um: golden-yellow
rubicundus, -a, -um: red
sēmen, sēminis n.: seed
mulier, mulieris f.: woman
exhibeō, -ēre, -uī, -itus: (here) give, provide
parturiō, -īre, -īvī, –: be in labor, be pregnant
prōmoveō, -ēre, -mōvī, -mōtus: advance; incite
partus, -ūs m.: offspring; birth, childbirth

Dissertātiō . . . Surinamēnsium: an expanded version of *Metamorphōsis Īnsectōrum Surinamēnsium*, the ornamental Latin title of the Dutch edition

Explicātiō Figūr. XLV.: *The Explanation for the 45th Figure*; Roman numerals ending in a period signify ordinals; see Figure 1 above for her illustration

planta novem pedum altitūdine crēscēns: *a plant growing to a height of nine feet*; a phrase in apposition to *Flōs Pāvōnis*; *altitūdine* is an ablative of degree of difference; the peacock flower (*Caesalpinia pulcherrima*) has been used in traditional medicine in Amazon tribes for centuries as a cure to some illnesses and as an abortifacient, as Merian will soon relate

ad prōmovendōs partūs labōrēs: this gerundive phrase set up by *ad* expresses purpose and is equivalent to the gerund construction *ad labōrēs partūs prōmovendum*; note that *partūs* is in the genitive

hinc fit ut Indae apud Belgās, in servitūte suā dūrius tractātae, eius ope sibi abortum accersant, līberōs tam gravī iugō sibi similēs & pariter mancipia nāscī praepedītūrae.

---

hinc: from this place; (here) hence
Indae, -ārum f.: Indian women (i.e. indigenous women)
apud (+ acc.): among
Belgae, -ārum m.: Belgians; Dutch (as the Low Countries used to collectively be labeled "Belgium")
servitūs, -ūtis f.: slavery
tractō (1): handle, treat
ops, opis f.: help
abortus, -ūs m.: abortion
accersō = arcessō, -ere, -īvī, -ītus: cause to come, elicit
līberī, -ōrum m.: children
iugum, -ī n.: yoke, bonds (of slavery)
similis, -e: similar
pariter (adv.): equally, likewise
mancipium, -iī n.: a taking by hand; property; (here) slave (cf. *e*mancipation)
nāscor, -ī, nātus sum: be born
praepediō, -īre, -īvī, -ītus: (here, with acc. and inf.) prevent (someone) from (some action)

---

fit . . . ut: an idiom for "it happens that," where *ut* introduces a noun clause that functions as the subject of *fit* and that takes the subjunctive *accersant*

dūrius tractātae: *treated rather harshly*; a circumlocution for sexual assault; *dūrius* is a comparative adverb from *dūrus, -a, -um*

eius ope: *through its* — the flower's — *help*; ablative of means

līberōs tam gravī iugō sibi similēs & pariter mancipia nāscī praepedītūrae: *to prevent their children from being born under so heavy a yoke similar to themselves and likewise as slaves*; *gravī iugō* is an ablative of attendant circumstance; a future participle (e.g. *praepedītūrae*, which modifies *Indae*) can express purpose or intention

Servae etiam Nigrītae, Surinamum ē Guinēā vel Angōlā dēlātae, nī benignē admodum excipiantur, neque prōcreant, neque expetunt īnfantēs: quīn mortem interdum sibi ipsīs cōnscīscunt, ob acerbitātem iugī, quō commūniter premuntur; in patriā enim sē ab amīcīs cum lībertāte regenerātum īrī arbitrantur; velutī hoc ipsae mihi ēnarrārunt.

---

serva, -ae f.: enslaved woman
Nigrītus, -a, -um: Black
Surinamum, -ī n.: Surinam (named after the Surinen people)
Guinēa, -ae f.: Guinea
Angōla, -ae f.: Angola
dēferō, -ferre, -tulī, -lātus: carry away
nī = nisi
benignē (adv.): kindly, benevolently
admodum (adv.): very
excipiō, -ere, -cēpī, -ceptus: take; (here) receive
prōcreō (1): bring forth (cf. procreate)
expetō, -ere, -īvī, -ītus: seek, long for
īnfāns, -antis m./f.: infant
quīn: (here, as adv.) indeed, in fact
interdum (adv.): occasionally
cōnscīscō, -ere, -scīvī, -scītus: decide; (here, with *mortem* and *sibi*) kill oneself
ob (prep. + acc.): on account of, because of
acerbitās, -ātis f.: harshness, severity
commūniter (adv.): jointly, universally
premō, -ere, pressī, pressus: press; oppress
patria, -ae f.: fatherland, homeland
lībertās, -ātis f.: liberty, freedom
regenerō (1): reborn (cf. regenerate)
arbitror, -ārī, -ātus sum: (here) believe
velutī = velut: just as, even as, like
ēnarrō (1): tell in detail, explain

---

etiam: links together the *servae . . . Nigrītae* with the *Indae* from earlier

Surinamum an accusative of place to which, where the preposition is left absent in the case of place names

nī . . . excipiantur: *unless they should be received . . .* ; the present subjunctive indicates a future-less vivid protasis

sē . . . regenerātum īrī: indirect statement introduced by *arbitrantur* with subject *sē* and future passive infinitive *regenerātum īrī*

ipsae: refers to the enslaved women

Ērūcae in hāc plantā vīvunt, caesiō colōre tīnctae, quae virentī eius fronde nūtriuntur. Annō 1700. diē 22. Iānuāriī quiētae iacentēs, mūtātā fōrmā aurēliae factae sunt colōris pūniceī; unde Februāriī diē 16. cineritiae phalaenae prōdiērunt, proboscide suā mel ē flōribus sūgentēs, quālem superius volantem dēlīneāvī.

ērūca, -ae f.: caterpillar
caesius, -a, -um: blue-grey (referring to eye color); (here) spotted
color, -ōris m.: color
tingō, -ere, tīnxī, tīnctus: wet; soak in color, paint
virēns, -entis: green (from *vireō, -ēre, -uī,* –, "to be green")
frōns, frondis n.: foliage
nūtriō, -īre, -īvī, -ītus: nourish, feed
Iānuārius, -iī m.: January (named after the Roman god of entrances, Janus)
quiēscō, -ere, quiēvī, quiētus: rest, be still
iaceō, -ēre, -uī, -itūrus: lie
mūtō (1): change
fōrma, -ae f.: form
aurēlia, -ae f.: chrysalis (the cocoon of a caterpillar; both terms derive from words meaning "gold," namely the Latin *aurum, -ī n.* and the Greek χρυσός, respectively)
pūniceus, -a, -um: reddish
unde: from which place, from which, wherefrom
Februārius, -iī m.: February
cineritius = cinericius, -a, -um: ash-grey
phalaena, -ae f.: moth (from the Greek word φάλλαινα, originally meaning "whale" or "monster")
prōdeō, -īre, -iī, -itus: come forth
proboscis, -idis f.: proboscis (i.e. the tubed, trunk-like mouth of an insect; from the Greek word προβοσκίς, "trunk")
mel, mellis n.: honey
sūgō, -ere, sūxī, sūctus: suck
quālis, -ē: of which kind, the sort of which
superius (adv.): farther above
volō (1): fly
dēlīneō (1): sketch out

---

caesiō colōre tīnctae: *caesiō colōre* is an ablative of means, and *tīnctae* is a PPP modifying *ērūcae*; Merian is likely describing the tobacco hornworm and hawk moth (*Manduca sexta*)

Annō 1700. diē 22. Iānuāriī: *In the 1700th year, on the 22nd day of January*; *annō* and *diē* are ablatives of time when

quiētae iacentēs: PPP and PAP, respectively, modifying the implicit subject of *factae sunt*, *ērūcae*

mūtātā fōrmā: ablative absolute

colōris pūniceī: genitive of description

proboscide suā: ablative of means

Explicātiō Figūr. XVIII.

Hāc Tabulā decimā & octāvā arāneās, formīcās & aviculās Belgīs *Colobritgens* dictās, in rāmō arboris *Guaiāvae* repraesentāvī, cum arāneae omnium maximae plērumque in istā arbore ā mē repertae sint.

---

tabula, -ae f.: (here) picture
decimus, -a, -um: tenth
octāvus, -a, -um: eighth
arānea, -ae f.: spider (cf. French "araignée")
formīca, -ae f.: ant (cf. French "fourmi")
avicula, -ae f.: little bird (diminutive form of *avis, -is f.*)
Colobritgens (indecl.): a Latinized name of the humming bird (cf. the French "Colibri" and German "Kolibri," or, in the diminutive, "Kolibritchen")
rāmus, -ī m.: branch
arbor, -oris f.: tree
Guaiāva, -ae f.: Latin name of the guava tree, of the genus *Psidium*
repraesentō (1): represent; (of artists) portray
plērumque (adv.): for the most part, mostly
reperiō, -īre, -perī (-erī), -ertus: find, discover

Hāc Tabulā decimā & octāvā: *through this eighteenth* (lit. "ten-and-eighth") *Picture*; ablative of means; see Figure 3 in this chapter for the artwork

Belgīs: dative of agent with the PPP *dictās*, which is used attributively

cum . . . repertae sint: causal *cum* clause with a perfect subjunctive verb



Figure 3: See the commentary on the next few pages for identifications of the species included in Figure XVIII. Image credit: *Metamorphosis Insectorum Surinamensium*, public domain.

In proximē sequentī figūrā XIX. iterum *Guaiāvam* cum īnsectīs suīs prōpositūra, dē eā nunc taceō, ad arāneās mē convertēns.

---

proximē (adv.): next; (here) immediately
iterum (adv.): again
prōpōnō, -ere, -posuī, -positus: put forth, display
taceō, -ēre, -uī, -itus: be silent, say nothing
convertō, -ere, -ī, -versus (+ *ad*): turn to

---

sequentī: although a PAP usually has *-ente* as its ablative singular ending, the more typical i-stem adjective ending *-entī* occurs when it is used as a non-verbal adjective (e.g. "following" with the sense of "next" dominating)

prōpositūra: *(being) about to display*; a future active participle modifying the subject of *taceō*, Merian herself (explaining the feminine gender)

eā: refers to *Guaiāvam*; in the description of figure XIX (see Figure 4), Merian specifies, among other features of the guava tree, its height ("as great as an apple tree in Germany," *Quantam in Germāniā mālus*), the color of its flowers ("milky-white with golden stamens," *lacteī . . . flāvēscentibus stāminibus*) and fruits ("a delicate sulfur-yellow rind . . . red flesh," *cortice teneriōre sulphureō . . . carnem . . . rubicundam*), its seeds, and its dispersion ("alone and in forests," *sponte & in sylvīs*)

ad arāneās mē convertēns: *turning (myself) to spiders*; the participle again modifies the subject of *taceō*



Figure 4: According to the curator's comments at the British Museum, the caterpillar forming the focal point of Figure XIX is a member of *Megalopyge lanata*. Merian cites and calls into question the ultimately false conjecture made by one of the earliest microbiologists, Antonie van Leeuwenhoek, that a caterpillar's tubercles (here, the red dots) are its eyes. Image credit: *Metamorphosis Insectorum Surinamensium*, public domain.

Istīusmodī magnās nigrāsque arāneās in *Guaiāvā* arbore plūrimās invēnī, quibus globōsus ille nīdus, tēlam ērūcae in sequentī figūrā exhibēns, praebet domicilium: nōn vērō, prout ā quibusdam peregrīnātōribus nōbīs obtrūdī coepit, longiōrēs tēlās dūcunt.

---

niger, -gra, -grum: black
plūrimus, -a, -um: most (superlative of *multus, -a, -um*)
inveniō, -īre, -vēnī, -ventus: find
globōsus, -a, -um: spherical
nīdus, -ī m.: nest
tēla, -ae f.: web (here, the cocoon of a caterpillar)
praebeō, -ēre, -uī, -itus: provide
domicilium, -iī n.: home (cf. domicile)
vērō (adv.): in truth; but, and (postpositive)
prout: just as, in the way that (from *prō* + *ut*)
peregrīnātor, -ōris m.: traveler
obtrūdō, -ere, -trūsī, -trūsus (+ dat.): (here, concerning false knowledge) force upon
coepī, -isse, -tus: begin
dūcō, -ere, dūxī, ductus: (here) produce, construct

---

istīusmodī: *of that kind*; a common merging of *istīus* and *modī*
quibus: *to whom*; dative of respect, like the subsequent *ērūcae*
in sequentī figūrā: i.e. figure XIX

Undique pilīs tēctae sunt, mūnītae īnsuper acūminātīs dentibus, quibuscum gravissimē morsitandō perīculōsa īnflīgunt vulnera, hīs ipsīs simul hūmōre quōdam īnfūsō. Commūnis ipsārum ēsca sunt formīcae, praeda, dum in arborēs excurrunt, haud facile effugiēns;

---

undique (adv.): from all sides; completely
pilus, -ī m.: hair
tegō, -ere, tēxī, tēctus: cover
mūniō, -īre, -īvī, -ītus: defend, arm; furnish, provide
īnsuper (adv.): in addition to, as well as
acūminātus, -a, -um: pointed
dēns, dentis m.: tooth
morsitō = morsicō (1): bite
perīculōsus, -a, -um: dangerous
īnflīgō, -ere, -flīxī, -flīctus: strike; inflict
vulnus, vulneris n.: wound
simul (adv.): at once, at the same time
hūmor, -ōris m.: liquid (here, venom)
īnfundō, -ere, -fūdī, -fūsus (+ acc. and dat.): pour into, infuse into
commūnis, -e: general, common
ēsca, -ae f.: food
praeda, -ae f.: (here, in the context of animals) prey
dum: (here) even while
excurrō, -ere, -cucurrī, -cursus: run forth
haud (adv.): not at all
facile (adv.): easily (adverbial form of *facilis, -e*)
effugiō, ere, -fūgī, -fugitūrus: flee away, escape

---

pilīs . . . acūminātīs dentibus: both are ablative of means

mūnītae: PPP modifying the implicit subject *arāneae*

quibuscum: equivalent to *cum quibus* and should be treated as an ablative of means

morsitandō: *by biting*; gerund and ablative of means

hīs ipsīs simul hūmōre quōdam īnfūsō: ablative absolute with the noun *hūmōre quōdam* and PPP *īnfūsō*; *hīs ipsīs* (referring to the *vulnera*) is a dative indirect object governed by *īnfūsō*

praeda . . . haud facile effugiēns: *a prey (that) escapes not at all easily*; a phrase in apposition to *formīcae*

quod mōre omnium arāneārum octō oculīs īnstrūctae sint, quōrum bīnī sūrsum, bīnī deorsum, bīnī ad dextram, bīnī dēnique ad sinistram spectant. Formīcārum dēfectū, ipsās ē nīdīs tollunt aviculās, omnemque corpusculīs cruōrem exsūgunt.

---

mōre (+ gen.): in the manner of, like
octō (indecl.): eight
īnstrūctus, -a, -um: furnished, provided (from *īnstruō, -ere, -strūxī, -strūctus*)
bīnī, -ae, -a: a pair, two
sūrsum (adv.): upwards
deorsum (adv.): downwards
dextra = dextera, -ae f.: right side
dēnique (adv.): finally
sinistra, -ae f.: left side
spectō (1): look, face
dēfectus, -ūs m.: (here) disappearance, absence
tollō, -ere, sustulī, sublātus: lift, raise; take away
corpusculum, -ī n.: little body (here of an animal, but in a philosophical context an atom)
cruor, -ōris m.: blood
exsūgō, -ere, -sūxī, -sūctus (+ dat.): suck out from

---

quod . . . octō oculīs īnstrūctae sint: *because . . . they are furnished with eight eyes*; *quod* here means "because," so it can take the subjunctive *sint*; *octō oculīs* is an ablative of means

Formīcārum dēfectū: ablative of attendant circumstance describing the situation in which spiders (like the pink-toe tarantula depicted, whose scientific name, adapted from the one provided by Linnaeus in 1758, is fittingly enough *Avicularia avicularia*) hunt hummingbirds

Pellem quidem parī modō ac ērūcae subinde mūtant; nunquam tamen volantēs reperī. Minus arāneōrum genus, quāle tēlae inhaerēns dēlīneātur, ōvula gerit sua sub ventre in placentā quādam, ubi etiam exclūduntur foetūs. Pariter octō oculīs mūniuntur, quī tamen hinc inde & cōnfūsē magis quam in istīs maiōribus locātī sunt.

---

pellis, -is f.: skin
pār, paris: equal, same
ac: (here, in a comparison) as
subinde (adv.): (here) now and then, frequently
nunquam = numquam (adv.): never
inhaereō, -ēre, -haesī, -haesus (+ dat.): cling to, hang on
ōvulum, -ī n.: little egg (diminutive of *ōvum, -ī*)
gerō, -ere, gessī, gestus: carry
venter, -tris m.: belly (cf. French "ventre")
placenta, -ae f.: (lit.) cake; disk (cf. placenta; from the Greek πλακόεις, -εσσα, -εν, "flat")
exclūdō, -ere, -clūsī, -clūsus: (here) hatch
foetus = fētus, -ūs m.: young, offspring
hinc (et) inde: here and there, in all directions
cōnfūsē (adv.): in disorder
locātus, -a, -um: placed, arranged (from *locō (1)*)

---

quidem parī modō ac ērūcae: *indeed in the same way as caterpillars*

quī: its antecedent is *octō oculīs* and introduces a relative clause with the verb *locātī sunt*

cōnfūsē magis quam in istīs maiōribus: *in more disorder than in those larger (spiders)*; *magis* is a comparative adverb modifying the adverb *cōnfūsē*, and the comparison uses *quam*; the smaller spider pictured in the top-left is likely an orb-weaver spider, famous for its circular webs that they destroy and rebuild daily — like other smaller spiders, they can even produce little silk kites to "balloon" through the air (first reported by Martin Lister, to whom Merian may be responding when she says that she never saw any spiders flying [Etheridge, 2011])

Praegrandēs in Amerīcā dantur formīcae, quae integrās arborēs ūnā nocte ita fronde suā nūdāre possunt, ut scōpās nōn arborēs referant. Bīnīs sunt armātae curvīs dentibus, forficis īnstar sē ipsōs intersecantibus;

---

praegrandis, -e: very large, very powerful

Amerīca, -ae f.: America (named after Amerigo Vespucci)

dō, dare, dedī, datus: (here, in the pass.) be said

integer, -gra, -grum: entire

nūdō (1): (here, with abl. of separation) strip of

scōpa, -ae f.: twig, branch

referō, -ferre, rettulī, relātus: bring back; report; (here) restore, produce again

armātus, -a, -um: armed, equipped (from *armō (1)*)

curvus, -a, -um: curved

forfex, forficis f.: scissors

īnstar (+ gen.): like

intersecō, -āre, -uī, -tus: (here) intersect

Praegrandēs . . . dantur formīcae: *ants are said (to be) very powerful*

ūnā nocte: ablative of time within which; here Merian is describing leafcutter ants, which can be a notorious agricultural pest in the region

ita . . . ut: *in such a way that*; *ita* anticipates a result clause

intersecantibus: the participle is in the ablative because it is modifying the ablative of means *curvīs dentibus*

hōrum ope sīc arboris resecant folia, ut in terram dēcidant, arbor vērō appāreat nūda, quālis in Eurōpā ab hyeme reddita cernitur. Dēcidentem in tellūre frondem mīllēnae excipiunt formīcae, hanc exspectantēs praedam, in nīdōs haud prō nūtrītiōne suā, sed tenellīs foetibus, adhūc vermiculīs, statim dēferendam.

---

resecō, -āre, -uī, -tus: cut off
folium, -iī n.: leaf (cf. French "feuille")
dēcidō, -ere, -ī, –: fall down
appāreō, -ēre, -uī, -itus: appear
nūdus, -a, -um: naked, bare (cf. nude)
Eurōpa, -ae f.: Europe (named after the mythical Europa)
hyems = hiems, hiemis f.: winter
reddō, -ere, -didī, -ditus: return; (here) render
cernō, -ere, crēvī, crētus: see (cf. discern)
tellūs, -ūris f.: earth (cf. tellurium)
mīllēnī, -ae, -a: thousand
exspectō (1): (here) look upon, consider
nūtrītiō, -ōnis f.: nutrition, nourishment
tenellus, -a, -um: tender; (more appropriately for gross larvae) young
adhūc (adv.): still
vermiculus, -ī n.: little worm, maggot (diminutive of *vermis, -is m.*; cf. vermin)
statim (adv.): at once

---

hōrum ope: *with the help of these (teeth)*; *ope* is an ablative of means

sīc . . . ut: *in such a way that*; like above, *sīc* anticipates a result clause

quālis in Eurōpā ab hyeme reddita cernitur: *the sort of which is seen in Europe to be rendered by the winter*; *reddita* is a predicate nominative with *cernitur*

praedam . . . dēferendam: *prey to be carried away*; *dēferendam* is a future passive participle modifying *praedam*

Perinde enim ac culicēs, sēmina pōnunt ālātae formīcae, unde vermiculī sīve acarī duplicis generis exclūduntur, aliīs eōrum tēlae sēmet involventibus, aliīs & plērīsque in nymphās trānseuntibus.

---

perinde (adv.): in the same manner
culex, culicis m.: gnat
pōnō, -ere, posuī, positus: (here, in the context of eggs) lay
ālātus, -a, -um: winged
sīve: or
acarus, -ī m.: mite (from the Greek word ἀκαρί)
duplex, duplicis: twofold
sēmet = sē
involvō, -ere, -ī, -volūtus (+ acc. and dat.): wrap with, envelop with
plērusque, -aque, -umque: most
nympha, -ae f.: nymph (from the Greek word νύμφη)
trānseō, -īre, -iī, -itus: (here) be transformed

---

sēmina: the Dutch edition reads "zaad," which also refers exclusively to seeds (i.e. plant "eggs"), and this word choice likely alludes to the visual similarity between seeds and ant eggs (as opposed to bird eggs)

aliīs eōrum . . . involventibus, aliīs & plērīsque . . . trānseuntibus: ablative absolutes with the nouns *aliīs* and *aliīs & plērīsque* and the respective participles *involventibus* and *trānseuntibus*; *aliīs eōrum . . . aliīs* should be translated as "some of them . . . others"

Hae nymphae illud ipsum sunt, quod reī ignārī formīcārum ōva nuncupant; cum haec multō sint minōra, illae ad nūtriendās Surinamī gallīnās faciant, meliusque hīs vel avēnā vel hordeō pābulum praebeant. Prōdeunt ex nymphīs formīcae, quae mūtātā pelle fiunt ālātae, & tum pōnunt ōvula, vermiculōs istōs prōdūcentia, quōrum nūtrītiōnī tantā alacritāte invigilant formīcae:

---

ignārus, -a, -um (+ gen.): ignorant (of)
ōvum, -ī n.: egg
nuncupō (1): call
multō (adv.): much
gallīna, -ae f.: hen
faciō, -ere, fēcī, factus (+ *ad*): be useful for
melior, -ius: better (comparative of *bonus, -a, -um*)
avēna, -ae f.: oats
hordeum, -ī n.: barley
pābulum, -ī n.: food
alacritās, -ātis f.: liveliness, eagerness, zeal
invigilō (1): watch over

---

illud ipsum . . . quod: *that very thing which*; *quod* introduces a relative clause with antecedent *illud*; *ipsum* is simply used for emphasis; the predicate *illud ipsum* does not match the number of the corresponding subject *hae nymphae*; Merian seems to criticize the Aristotelian proposition of William Harvey that the nymphs of insects should be considered eggs, specifically as a second step in a larva's development from a spontaneously generated "imperfect" egg to its perfected adult state (Reynolds, 2019)

cum haec multō sint minōra, illae ad nūtriendās Surinamī gallīnās faciant: *since the latter are much smaller, the former should be useful for feeding the hens in Surname*; the neuter plural *haec* ("the latter") refers to *ōva* while the feminine plural *illae* ("the former") refers to *nymphae*; *nūtriendās Surinamī gallīnās* is a gerundive phrase equivalent to *gallīnās Surinamī nūtriendum*, where *Surinamī* is a locative; *sint* is a subjunctive in a causal *cum* clause, while *faciant* is a hortatory subjunctive

meliusque hīs vel avēnā, vel hordeō, pābulum praebeant: *and they should provide better food to these (hens) than either oats or barley*; *melius* is a comparative adjective modifying *pābulum* and *avēnā* and *hordeō* are ablatives of comparison; *praebeant* is again a hortatory subjunctive

mūtātā pelle: *with the skin having been changed*; ablative absolute

quōrum nūtrītiōnī tantā alacritāte invigilant formīcae: the antecedent of *quōrum* is *vermiculōs*; the compound verb *invigilant* takes the dative *nūtrītiōnī*; *tantā alacritāte* is an ablative of manner; leafcutter ants actually do not feed the collected leaves to their larvae, but rather to a fungus (*Leucoagaricus gongylophorus*) that they cultivate in a mutualistic relationship as food for their young

in calidīs etenim illīs regiōnibus adversus frīgora futūra nōn opus est quod sibi prōspiciant, quoniam nūlla unquam ibi rēgnat hyems. Cavernās in terrīs excavant ad octō pedum altitūdinem, tam concinnē effōrmātās, ut hūmānae artis opus dīcerēs.

---

calidus, -a, -um: hot
etenim: for (usually introducing a self-evident statement)
regiō, -ōnis f.: region
adversus (adv.): against
frīgus, -oris n.: coldness; (pl.) the cold
futūrus, -a, -um: future, coming (from the future active participle of *esse*)
opus est: it is necessary
prōspiciō, -ere, -spexī, -spectus: (intrans. with dat.) look out for, take care of
quoniam: since
unquam = umquam (adv.): ever
ibi (adv.): there
rēgnō (1): rule
caverna, -ae f.: cavern
excavō (1): hollow out, excavate
concinnē (adv.): beautifully, elegantly
effōrmō (1): shape
hūmānus, -a, -um: human
ars, artis f.: art
opus, operis n.: work

---

adversus frīgora futūra: take closely with *sibi prōspiciant*

quod sibi prōspiciant: *quod* introduces a noun clause, which grammatically functions as the subject of *opus est*, but it can be taken together with *opus est* as "it is necessary that"

effōrmātās: a PPP modifying *cavernās*

tam . . . ut . . . dīcerēs: *so . . . that you might call (it)*; *tam* anticipates a result clause; *dīcerēs* is imperfect because the result clause is subordinate to a participial clause in the perfect tense

Quod sī aliōrsum trānsmigrāre cupiunt, nōn datā ad eōusque perveniendum viā, pontem hōc sibi struunt modō, ut prīma alicubi dentibus lignō īnfīxīs quiēscat, secunda priōrem sequātur eīque adhaereat firmiter, tertia rūrsus secundae, quārta dēnuō tertiae ⇒

---

quod sī: but if
aliōrsum (adv.): elsewhere
trānsmigrō (1): migrate
cupiō, -ere, -īvī, -ītus: want
eōusque (adv.): all the way to that place (from *eō* + *usque*)
perveniō, -īre, -vēnī, -ventus: come to, arrive at
pōns, pontis m.: bridge
struō, -ere, strūxī, strūctus (+ acc. and dat.): make by joining (something) together
alicubi (adv.): somewhere
lignum, -ī n.: wood
īnfīgō, -ere, -fīxī, -fīxus (+ dat.): drive in, fasten in
prior, prius: first, previous
adhaereō, -ēre, -haesī, -haesus (+ dat.): cling to, hang onto
firmiter (adv.): firmly
rūrsus (adv.): again
dēnuō (adv.): again, once more

---

nōn datā ad eōusque perveniendum viā: ablative absolute with the noun *viā* and the PPP *datā*; notice the use of the (accusative) gerund with *ad* to express purpose in *ad eōusque perveniendum*; Merian is now writing about a different kind of ant, the army ant, which, during its recurring raids, can completely freeze its body and provide an attachment point for other ants that follow; using this process, army ants create not only short bridges, but also large, spherical nests called "bivouacs"

hōc . . . modō, ut: *in the following way, namely*; in later Latin, *hic, haec, hoc* is often used to refer to a subsequent clause, often a noun clause introduced by *ut* (here with the subjunctive verbs *quiēscat*, *adhaereat*, *expōnant*, and *īnserviant*)

prīma . . . secunda priōrem . . . tertia . . . secundae, quārta . . . tertiae: all the ordinals in this clause modify an implicit *formīca* in the appropriate case

dentibus lignō īnfīxīs: ablative absolute with subject *dentibus* and PPP *īnfīxīs*

(continued) & ita sibi ipsīs porrō invicem appendentēs līberē sēsē expōnant ventō quousque huius flātū ad istum locum prōiiciantur, & sīc mīllēnīs aliīs trānscurrentibus prō ponte īnserviant.

---

porrō (adv.): (here) further
invicem (adv.): in turn, one after another
appendō, -ere, -ī, -pēnsus: hang on (cf. append)
līberē (adv.): freely
sēsē: emphatic version of *sē*
expōnō, -ere, -posuī, -positus (+ acc. and dat.): expose to
ventus, -ī m.: wind
quousque: until (from *quō* + *usque*, "all the way to which point/time")
flātus, -ūs m.: blowing, breeze
prōiiciō = prōiciō, -ere, -iēcī, -iectus: fling away
trānscurrō, -ere, -cucurrī, -cursus: go over, run across
īnserviō, -īre, -īvī, -ītus (+ dat.): be serviceable to

---

appendentēs: a PAP modifying the subject of *expōnant*

huius: refers to *ventō*

prōiiciantur: *quousque* in this case takes a subjunctive verb because this clause has a close logical connection to the surrounding noun clause and its subjunctive verbs

Cum arāneīs, reliquīsque omnibus huius terrae īnsectīs, perpetua hīs formīcīs intercēdit inimīcitia. Quandō ē cavernīs suīs ērumpunt, id quod singulīs annīs fit semel, innumera eārum prōdit multitūdō, quā domōs replent, ex ūnō in aliud conclāve trānsmigrant, & animalcula cūncta maiōra, minōra, exsūgendō necant.

---

reliquus, -a, -um: remaining; other
perpetuus, -a, -um: perpetual
intercēdō, -ere, -cessī, -cessus: exist between
inimīcitia, -ae f.: enmity (from *in-* "not" + *amīcitia* "friendship")
quandō: when
ērumpō, -ere, -rūpī, -ruptus: burst forth, break out
singulī, -ae, -a: individual
semel (adv.): once
innumerus, -a, -um: innumerable
multitūdō, -inis f.: multitude
repleō, -ēre, -ēvī, -ētus: fill up
conclāve, -is n.: room, chamber
animalculum, -ī n.: little animal (diminutive of *animal, -ālis n.*)
cūnctus, -a, -um: whole, entire
necō (1): kill

Cum arāneīs . . . perpetua hīs formīcīs intercēdit inimīcitia: *intercēdere alicui cum aliō* is an idiom for "exist between one person and another"

id quod singulīs annīs fit semel: *id quod*, "a thing which," can be used in place of the lone *quod* when referring and in apposition to an entire previous clause; *singulīs annīs* means "every year"

ex ūnō in aliud conclāve: *from one (room) into another room*; include an implicit *conclāvī* which the ablative *ūnō* modifies

exsūgendō: *by sucking*; gerund and ablative of means

Maius istīusmodī phalangium mōmentō ferē hinc est cōnsūmtum, quoniam tantīs cumulātur acervīs ut expedīre tam citō sēsē & ā perīculō līberāre nequeat.

---

phalangium, -iī n.: venomous spider, tarantula (from the Greek φαλάγγιον, lit. "a little phalanx")
mōmentō (adv.): in a moment, in an instant (from *mōmentum, -ī n.*)
ferē (adv.): nearly, almost
cōnsūmō, -ere, -psī, -ptus (-tus): consume
cumulō (1): pile on; overwhelm
acervus, -ī m.: heap, pile
expediō, -īre, -īvī, -ītus: set free, extricate
citō (adv.): quickly
perīculum, -ī n.: danger
līberō (1): free, liberate
nequeō, -īre, -īvī, -itus: cannot

---

istīusmodī: *of that kind*; a common merging of *istīus* and *modī*

est cōnsūmtum: should be translated as *est* with a PPP (i.e. "is consumed" instead of the perfect passive indicative "was consumed"), as is quite common in post-Classical works

tantīs . . . acervīs, ut: *by heaps so great that*; ablative of means; *tantīs* anticipates the following result clause

Quīn ipsī hominēs fugā sibi caveant oportet; adeō ūnum post aliud acervātim permeant cubiculum, atque sīc tōtā domō vāstātā & quasi tersā, proximam petunt, dōnec tandem ad suās redeant caveās.

---

fuga, -ae f.: flight; (more generally) swift motion, rush

caveō, -ēre, cāvī, cautus (+ *sibi* and abl.): be on one's guard against, be wary of

oportet, -ēre, -uit, –: (impers.) it is necessary that

adeō (adv.): (here) for, indeed

acervātim (adv.): in heaps

permeō (1): pass through (cf. permeate)

cubiculum, -ī n.: room

vāstō (1): make empty, desert

quasi: (here) almost

tergeō, -ēre, tersī, tersus: wipe clean; clear

proximus, -a, -um: nearest; (here) next

petō, -ere, -īvī, -ītus: seek; (here) attack, assail

dōnec: until

tandem: at last, finally

redeō, -īre, -iī, -itus: return

cavea, -ae f.: cave

---

caveant: *oportet* can set up an accusative-infinitive construction, a noun clause with *ut* and a subjunctive verb, or even a subjunctive verb alone

ūnum post aliud . . . cubiculum: *one room after another*

tōtā domō vāstātā & quasi tersā: ablative absolute with the noun *domō* and the PPPs *vāstātā* and *tersā*

proximam: modifies an implicit *domum*

redeant: in later Latin, *dōnec* regularly takes the subjunctive, which in this case likely communicates the uncertainty in the length of a rush

Aviculae *Colobritgens* Belgīs dictae ab arāneō nīdīs suīs īnsidentēs capiuntur; aliās cibus Sacerdōtum Surinamēnsium, quibus, ceu relātum mihi est, aliīs vēscī cibīs prohibitum est. Quaterna perinde ut aliae avēs pōnunt ōva, eaque incubandō exclūduntur.

īnsideō, -ēre, -sēdī, -sessus (+ dat.): sit on
aliās (adv.): at other times
cibus, -ī m.: food
sacerdōs, sacerdōtis m.: priest
ceu: like, as
vēscor, -ī, – (+ abl.): eat
prohibeō, -ēre, -uī, -itus: prohibit
quaternī, -ae, a: four at a time
avis, -is f.: bird
incubō (1): lie on; sit upon, brood (cf. incubate)

Aviculae *Colobritgens* Belgīs dictae: Merian is speaking of not just any "little birds," but specifically those "(that are) called *Colobritgens* by the Dutch"; *Belgīs* is a dative of agent with the PPP *dictae*

nīdīs suīs īnsidentēs: *while sitting on their own nests*; circumstantial participial phrase

aliās cibus Sacerdōtum Surinamēnsium: assume the verb *sunt*, which has *aviculae* as its implicit subject and *cibus* as a predicate

quibus . . . vēscī . . . prohibitum est: *for whom eating . . . is prohibited*; *quibus* is a dative of reference; *vēscī* is the subject infinitive of *prohibitum est*

Quaterna: modifies *ōva*; four is an unusually large number of eggs for a hummingbird, with two being more common

incubandō: *through brooding*; ablative of means

Celerrimē volitant aviculae, expānsīsque mel ē flōribus sūgunt ālīs, ut immōtae velutī in āere haereant, variīs ēlegantissimē pictae colōribus ipsōque pāvōne pulchriōrēs.

---

celer, -is, -e: quick
volitō (1): fly to and fro, flit about
expandō, -ere, -ī, -pānsus: spread out
āla, -ae f.: wing
immōtus, -a, -um: motionless
āēr, āeris n.: air (from the Greek ἀήρ)
haereō, -ēre, haesī, haesus (+ *in*): adhere, stick to, remain fixed in
varius, -a, -um: various
ēlegāns, -antis: elegant
pingō, -ere, pīnxī, pictus: paint, picture

---

expānsīsque . . . ālīs: ablative absolute with PPP *expānsīs*; hummingbird wings are particularly remarkable as they can rotate in a figure-8 cycle, completed roughly 75 times per second; this unique ability not only allows hummingbirds to fly faster than any other bird relative to their miniscule size (up to 35 mph), but also hover in place while sucking nectar with their curved beak; however, they also spend a lot of energy maintaining this flight, requiring them to feed almost constantly

pictae: PPP modifying the subject of *haereant*

ipsōque pāvōne pulchriōrēs: *pulchriōrēs* is a comparative adjective and *pāvōne* an ablative of comparison; one notable visual similarity between peacocks and hummingbirds is their iridescent plumage

Chapter 7 — Newton: *Dē Mundī Systēmate*

Almost every schoolchild is familiar with the legend of the falling apple that eventually inspired the gravitational theory of Isaac Newton (1643 CE[1] – 1727 CE), which was first recounted in writing in the fifteenth letter of *Letters on the English* by the renowned French writer Voltaire. He framed the story as an example of Newton's scientific thought process, in which he would observe a well-known phenomenon, such as the constant acceleration of an object dropped not far above the surface of the Earth, and then would generalize the associated principles to all bodies in the universe (Voltaire, 1894). Newton's theory of universal gravitation replaced previous attempts at bridging the seemingly arbitrary divide between terrestrial and celestial motions in the Aristotelian worldview and Copernican planetary model (as well as at addressing the lack of explanatory power in the latter). The most influential of these earlier proposals came from the French philosopher René Descartes, who believed that all changes could be reduced to local collisions between tiny (infinitely divisible) corpuscles that compose matter. According to this view, vortices of aether circling any attractive body like the sun uphold the planetary motions, at the time considered circular, with these particles pushing the planets from either side in just the right proportions. Despite the qualitative simplicity that made this description of gravity so attractive to other French physicists (beyond mere patriotic loyalty), Descartes's explanation lacked a quantitative element that would have made it usable in practice. It would also turn out to be inconsistent with the elliptical orbits discovered by Kepler (Westfall, 1971). Voltaire therefore declared Newton's ap-

[1] According to the Julian calendar used at the time in England, Newton was famously born on Christmas Day of 1642, the year that Galileo Galilei died. However, his birthyear is usually given as 1643 because the Gregorian calendar was already in use across the rest of Europe, even if it was only adopted in the British Isles in 1752.

proach superior: It not only resolved the discrepancies with Kepler's laws, but it also provided a new outlook on science by introducing the concept of attractive forces, thereby establishing what would later be known as Newtonian physics.

The creation of Newtonian physics was associated with the discovery of calculus or, per Newton's own terminology, "the method of fluxions" (*Methodus Flūxiōnum*), which was simultaneously conceived of by Leibniz, leading to a life-long conflict of attribution. Considering that calculus has become a *sine quā nōn* of high-school-level kinematics courses (due to the use of derivatives in defining velocity and acceleration), it may be surprising that Newton refrained from employing his wonderful new tool in his revolutionary work titled the *Philosophiae Nātūrālis Prīncipia Mathēmatica*. Instead, he exclusively formulated his results and proofs by means of Euclidean geometry so that his contemporaries could appreciate the text without a significant novel mathematical background. Still, he did end up providing a crash course on a simplified version of calculus involving geometrical limits, also known as "first and last ratios," in Book 1 (González-Velasco, 2011). In true Euclidean fashion, Newton's axioms, the three laws of motion, form the foundation of all his subsequent derivations in Book 1, including proofs of Kepler's three laws of planetary motion (see page 104) and the Shell Theorem, which allows a perfectly spherical gravitational body to be represented by a point mass. The second book shifts the focus to mediums of fluids and forces of resistance, a decision that some scholars have interpreted as an arguably tangential attempt at demonstrating the erroneousness of Descartes's vortex theory. Book 3 concludes the *Prīncipia* by presenting a direct application of Newton's gravitational theory to the motions of the planets and even the moons

of Jupiter, which had been discovered less than a century earlier by Galileo Galilei (Smith, 2024).

However, Newton initially did not intend to distribute the topics across three volumes, having previously envisioned only two: one devoted to all the calculations, which eventually transformed into Book 1 of the *Prīncipia*, retaining the title *Dē Mōtū Corporum*, and another aimed at a wider audience. A remnant of this early draft of the second part survives through the fairly short posthumous treatise *Dē Mundī Systēmate Liber* (frustratingly enough the same title as Book 3), which has a somewhat confounding history. The text, in the Latin of the original manuscript authored by Newton, was technically first made available to the public by John Conduitt in 1728, the year following the physicist's death. Coincidentally, in the exact same year, an anonymous editor, frequently assumed to be Andrew Motte, independently published an English translation with the title *Treatise of the System of the World*, containing a not insignificant number of disagreements with the Latin version. The most noticeable difference was the removal of references to specific prepositions, as the numbering obviously differed from the finalized editions of the *Prīncipia* — they were restored in a second and now definitive edition in 1731, consistent with the numbering of the 1729 English translation of the *Prīncipia*. According to the preface of the first edition of the *Treatise*, the decision to publish what Newton would have intended to be a Latin work in English was motivated by a desire to "render it more useful to our countrymen" (Cohen, 2004). While the comment may perhaps be suggestive of the beginning of an end for Latin as the international language for science, it is certainly in line with Newton's original goal of communicating and popularizing his new worldview in an easily understandable manner.

The excerpt below, from the first few pages of *Dē Mundī Systēmate*, exemplifies this objective of scientific accessibility. It introduces the now well-known thought experiment of Newton's cannonball (referred to only as a "projectile"), which serves as an extension of the aforementioned tale of the apple by explaining, in simple and visually appealing terms, the equivalence between a thrown or falling body and an orbiting one. At a local scale where the curvature of the Earth is negligible, the gravitational force on the cannonball from the center of the Earth is directed downward and appears constant. As Galileo had suspected even prior to experimentally determining that the displacement of a falling body is proportional to the square of time (Capecchi, 2014), the ball traces a parabolic arc. Newton's innovation lies in noting that, if the cannonball repeatedly overshoots the surface of the Earth, its trajectory is ultimately circular or elliptical, as required by Kepler's first law. Since the equations dependent on the mass and velocity of the projectile arising from the theory of gravity are quadratic in nature, all of these different cases correspond to conic sections, curves first considered by Ancient Greek geometers such as Apollonius. In fact, the cannonball thought experiment can even be generalized to account for the paths of the historically portentous comets: While the similarly ominous eclipses had been predicted and understood ever since Thales and Anaxagoras in the 6th and 5th centuries BCE, comets lacked a proper mathematical treatment until Newton suggested in Book 3 of the *Prīncipia* that they move elliptically or even parabolically, in the same way a sufficiently speedy cannonball that escapes Earth can follow a parabola, never to return, or simply a large ellipse. Edmund Halley later used Newtonian physics and observational data to successfully predict the reappearance of the eponymous Halley's comet in 1758, at least

within a margin of error due to the gravitational influences of Jupiter and Saturn (Campbell, 1909).

Indeed, later approaches for tackling more complex scenarios than the Earth-cannonball system, such as the infamous three-body problem (which, in an instance of chaos theory, cannot be solved analytically in the general case), were vital for the 19th-century discovery of Neptune prompted by the detection of irregularities in the orbit of Uranus. Despite the great empirical success of Newtonian physics, potential holes began to emerge, including the gradually increasing divergence between the precession of Mercury's orbit from the predicted value, a phenomenon ultimately explained by Newtonian physics's 20th-century successor, general relativity (Fernie, 1994).

Source Text: Our Latin text comes from Newton (1728).

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Dē Mundī Systēmate Liber
Effectūs vīrium centripetārum

Vīribus centripetīs Planētās in orbibus certīs retinērī posse intelligētur ex mōtibus prōiectilium. Lapis prōiectus urgente gravitāte suā dēflectitur dē cursū rēctilīneō, & curvam līneam in āere dēscrībendō tandem cadit in terram;

---

mundus, -ī m.: world, universe (cf. French "monde")
systēma, -atis n.: system (from the Greek σύστημα, something "put together")
effectus, -ūs m.: effect
vīs, vīs f.: (here, in physics) force
centripetus, -a, -um: centripetal
planēta, -ae f.: planet (derived from the Ancient Greek πλάνητες ἀστέρες, meaning "wandering stars")
orbis, -is m.: orb, sphere; (here) orbit
certus, -a, um: fixed
retineō, -ēre, -uī, -tentus: hold back, keep
intelligō = intellegō, -ere, -lēxī, -lēctus: understand
mōtus, -ūs m.: movement, motion
prōiectile, -is n.: projectile (i.e. the cannonball; neuter substantive of *prōiectilis, -e*, "able to be thrown")
lapis, -idis m.: stone (cf. lapis lazuli)
prōiiciō = prōiciō, -ere, -iēcī, -iectus: throw
urgeō, -ēre, ursī, –: press, bear down
gravitās, -ātis f.: weight; (in physics) gravity
dēflectō, -ere, -flexī, -flexus: turn aside, deflect
cursus, -ūs m.: course
rēctilīneus, -a, -um: rectilinear, straight
curvus, -a, -um: curved
līnea, -ae f.: line
āēr, āeris m.: air (from the Greek ἀήρ)
dēscrībō, -ere, -scrīpsī, -scrīptus: (here) delineate, describe
tandem: at last, finally
cadō, -ere, cecidī, cāsus: fall

Vīribus centripetīs: ablative of means; the term "centripetal force," first used by Newton himself, describes a force "seeking" or acting towards the "center"

intelligētur: *it will be understood that*; an impersonal verb that introduces the indirect statement with subject *Planētās* and infinitive *posse*

urgente gravitāte suā: another ablative of means

curvam līneam in āere dēscrībendō: *by describing a curved line in the air*; this gerund serves as an ablative of means and takes an accusative object

sī mōtū vēlōciōre prōiiciātur, pergit longius. Augendō vēlōcitātem, fierī posset ut arcum dēscrīberet mīlliāris ūnīus, duōrum, quīnque, decem, centum, mīlle; ac tandem ut, pergendō ultrā terminōs Terrae, nōn amplius in terram caderet.

---

vēlōx, -ōcis: swift
pergō, -ere, perrēxī, perrēctus: go; (+ inf.) proceed to
longius (adv.): longer, farther (comparative of *longē*)
augeō, -ēre, auxī, auctus: increase (cf. auction)
vēlōcitās, -ātis f.: velocity
arcus, -ūs m.: bow; arc
mīlliāre, -is n. = mīliārium, -iī n.: mile-stone; mile
decem (indecl.): ten
centum (indecl.): hundred
ultrā (prep. + acc.): beyond
terminus, -ī m.: boundary, end
nōn amplius: no longer

---

sī mōtū vēlōciōre prōiiciātur: a future-less-vivid conditional with a present subjunctive verb that is usually translated with "should"; the apodosis, however, has a much more definite force thanks to the indicative verb *pergit*, because, while the initial speed of the cannonball is variable, a greater speed will *necessarily* result in a greater distance traveled

Augendō vēlōcitātem: gerund with an accusative object acting as an ablative of means

fierī . . . ut: an idiom for "it happens that"; specifically, *ut* introduces a noun clause that functions as the subject of *fierī* and that takes the subjunctives *dēscrīberet* and *caderet*, which are imperfect due to the sequence of tenses (indeed, *posset* is of the secondary sequence)

posset: an imperfect potential subjunctive that can be expressed as "could" or, with *fierī*, "would be possible"

mīlliāris ūnīus, duōrum, . . . mīlle: *of one mile, (or) two, . . . (or) a thousand*; a genitive of description

pergendō ultrā terminōs Terrae: a gerund functioning as an ablative of means

Dēsignent AFB superficiem terrae, C centrum eius, & VD, VE, VF, līneās curvās quās prōiectile dē montis praealtī vertice V, secundum līneās horizontī parallēlās auctīs cum vēlōcitātis gradibus, successīvē ēmissum dēscrībat. Et nē āeris resistentia, quā mōtūs coelestēs vix retardantur, in computum veniat, fingāmus hunc omnem tollī, vel saltem nīl resistere.

---

dēsignō (1): designate, represent
superficiēs, -ēī f.: surface
centrum, -ī n.: center (from the Greek κέντρον)
praealtus, -a, -um: very high
vertex, verticis n.: top
secundum (prep. + acc.): with respect to, following (in the context of spatial relations)
horizōn, horizontis m.: horizon (from the Greek ὁρίζων)
parallēlus, -a, -um (+ dat.): parallel to (from the Greek παράλληλος, -ον)
gradus, -ūs m.: (here) degree
successīvē (adv.): successively, in stages
ēmittō, -ere, -mīsī, -missus: send forth
resistentia, -ae f.: resistance
coelestis = caelestis, -e: celestial
vix (adv.): scarcely, hardly
retardō (1): hinder, delay, retard
computus, -ī m.: computation
fingō, -ere, fīnxī, fictus: (here) pretend (cf. feign, fiction)
tollō, -ere, sustulī, sublātus: lift, raise; (here) remove
saltem (adv.): at least
nīl = nihil
resistō, -ere, -stitī, –: resist

Dēsignent: *let . . . represent*; the subjects of this jussive subjunctive are the lengths and points *AFB*, *C*, *VD*, *VE*, and *VF*; see Figure 1 for the diagram

quās prōiectile . . . dēscrībat: *which a projectile may describe*; the subjunctive indicates hypotheticality (indeed, *VD*, *VE*, *VF* are merely examples of the path of a projectile)

ēmissum: a PPP modifying *prōiectile* and bracketing a long participial phrase

nē . . . veniat: a negative purpose clause

hunc omnem: modifies an implicit *āerem*

Newton's Cannonball

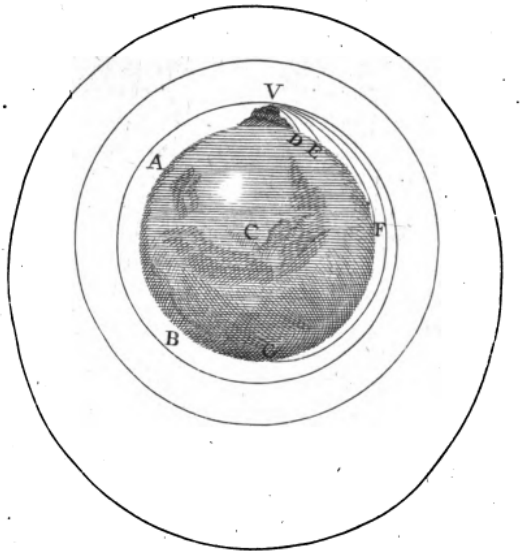


Figure 1: This image accompanies Newton's cannonball thought experiment in the *Dē Mundī Systēmate* — it has become so recognizable that a photograph of the corresponding page in the second English edition is included in the Voyager Golden Record (a collection of sounds and pictures intended to introduce humanity to extraterrestrials) under the name "page of book." It is interesting that Newton does not consider landing points past *G*, because it is impossible for an object to land farther than the point diametrically opposite to the mountain. Indeed, if the Earth were collapsed to a point mass, as permissible by the Shell Theorem, Kepler's first law shows that the cannonball will travel on an ellipse with the Earth at one focus and the mountain as an apsis. Such an ellipse intersects the Earth's surface twice, and the first intersection the cannonball reaches, the landing point, is always before point *G*, at least if one assumes, as Newton does, that the cannon is pointed horizontally (Pesnell, 2018). Image credit: *A Treatise of the System of the World*, public domain.

Et eādem ratiōne quā corpus vēlōcitāte minōre dēscrībit arcum minōrem VD, & maiōre arcum maiōrem VE, & auctā adhūc vēlōcitāte pergit longius ad F, & longius ad G, idem tandem, sī augeātur semper vēlōcitās, superābit tōtum tellūris ambitum, & redībit ad montem unde fuerat prōiectum.

---

ratiō, -ōnis f.: reason
adhūc (adv.): (here) still
superō (1): surpass
tellūs, -ūris f.: Earth (cf. tellurium)
ambitus, -ūs m.: (here) circumference
redeō, -īre, -iī, -itus: return
unde: from which place, from which, wherefrom

---

eādem ratiōne quā: *for the same reason that*; ablatives of cause; *quā* introduces a relative clause with subject *corpus* and verbs *dēscrībit* and *pergit*

corpus vēlōcitāte minōre . . . & maiōre: *a body with less velocity . . . and with greater (velocity)*; ablatives of description for *corpus*

auctā adhūc vēlōcitāte: *with the velocity having still been increased* or *(a body) with a velocity still increased*; can be interpreted as either an ablative absolute with PPP *auctā* or another ablative of description for *corpus*

idem: modifies an implicit *corpus* and serves as the subject of *superābit*

sī augeātur semper vēlōcitās: the future-less-vivid protasis of another mixed conditional

unde fuerat prōiectum: *unde* functions like a relative pronoun here; *prōiectum fuerat* is equivalent to the pluperfect passive *prōiectum erat*

Cumque ārea, quam radiō ad centrum terrae ductō dēscrībit, sit (per Prōp. 1. lib. 1. Prīncip. Math.) prōportiōnālis temporī, vēlōcitās eius in reditū ad montem nōn minor erit quam sub initiō: servātā autem vēlōcitāte potest idem saepius eādem lēge revolvī.

ārea, -ae f.: (in math) area
radius, -iī m.: (in math) radius
dūcō, -ere, dūxī, ductus: (here) draw
Prōp. = prōpositiō, -ōnis f.: proposition
lib. = liber
Prīncip. Math.: *Philosophiae Nātūrālis Prīncipia Mathēmatica* (see the introduction)
prōportiōnālis, -e (+ dat.): proportional to
reditus, -ūs m.: return
sub (prep. + abl.): (here with time) at
initium, -iī n.: beginning
servō (1): preserve
saepius (adv.): more often; repeatedly (comparative of *saepe*)
lēx, lēgis f.: law (of government and nature)
revolvō, -ere, -ī, -volūtus: revolve; return

Cumque . . . sit: a causal *cum* clause with a subjunctive verb

quam radiō ad centrum terrae ductō dēscrībit: the implicit subject is *corpus*; *radiō . . . ductō* is an ablative of means; in other words, the area is enclosed by the orbit of the cannonball and the lines from the cannonball to the center of the Earth at two different times

per Prōp. 1. lib. 1. Prīncip. Math.: also known as Kepler's second law, which, along the lines of the formulation given by Newton here, states that the area covered by the radius is a linear function of the time elapsed from some chosen starting point; in fact, Newton showed that it applies to any body experiencing forces acting towards a single, central point (be it gravity or any other force, not necessarily proportional to the distance squared)

vēlōcitās eius . . . nōn minor erit quam sub initiō: *its velocity . . . will not be less than (the velocity) at the beginning*; there is an implicit *vēlōcitās* after *quam*, which is used for a comparison; *initiō* is an ablative of time when

servātā . . . vēlōcitāte: ablative absolute

idem: again modifies an implicit *corpus*

eādem lēge: ablative of cause (refers to Kepler's second law)

revolvī: *return*; the passive form of a transitive verb of motion expresses a reflexive, middle voice; see the note in the introduction to Copernicus for more detail

Imāginēmur iam corpora dē regiōnibus altiōribus secundum līneās horizontālēs prōiicī, puta dē locīs mīlliāria quīnque, decem, centum, mīlle vel plūra, totidemve Tellūris sēmidiametrōs, altīs; & prō variā corporum vēlōcitāte & vī gravitātis in singulīs regiōnibus exercitā, dēscrībentur arcūs, Tellūrī vel concentricī vel variē excentricī; inque hīs trāiectōriīs pergent corpora ad modum Planētārum coelōs trānscurrere.

---

imāginor, -ārī, -ātus sum: imagine
regiō, -ōnis f.: region
horizontālis, -e: horizontal
puta (adv.): for instance, namely (from the imperative form of *putō (1)*)
plūs, plūris: more (comparative of *multus, -a, -um*; in the singular, usually only neuter)
totidem (indecl.): just as many
-ve: or
sēmidiametros, -ī m.: semidiameter, radius
varius, -a, -um: various, varying
singulī, -ae, -a: individual
exerceō, -ēre, -uī, -itus: work; (in physics) exert (a force)
concentricus, -a, -um (+ dat.): concentric to
excentricus = eccentricus, -a, -um (+ dat.): eccentric to
trāiectōria, -ae f.: trajectory
ad modum (+ gen.): in the manner of, like
coelum = caelum
trānscurrō, -ere, -cucurrī, -cursus: go over, run across

---

mīlliāria . . . sēmidiametrōs: notice the bracketing, with these accusatives of distance or extent of space lying between the noun-adjective pair *locīs . . . altīs*

exercitā: PPP modifying *vī* and concluding a participial phrase

coelōs: the plural forms of *caelum* are masculine and are a poetic way to say "the heavens"

Chapter 8 — Euler: "Solūtiō Problēmatis ad Geōmetriam Sitūs Pertinentis"

Leonhard Euler (1707 CE – 1783 CE) holds the title of most prolific mathematician in history, with his only competitor in terms of the number of papers being the highly collaborative 20^{th}-century mathematician Paul Erdős. The Wikipedia page devoted to theorems, equations, and other mathematical subjects named after Euler — usually with a second discoverer credited, for the sake of distinguishability — has 54 entries, ranging from differential equations describing the flow of liquids to the beloved identity $e^{i\pi} + 1 = 0$, which in turn contains Euler's number $e \approx 2.718$. The sheer volume of his output, which the Swiss Academy of Sciences has been attempting to compile into an *Opera Omnia* since 1907 with the help of an index system known as the Eneström number (Kleinert, 2015), is made all the more impressive by the fact that, for much of his life, Euler suffered from poor eyesight, brought upon by illness and allegedly worsened by his perusing of data tables. Following an operation with only temporary success in 1771, he became almost entirely blind, an impairment that, he mused, led to "one less distraction" (Calinger, 2007). Even in his early studies Euler was well-connected with major contemporary mathematicians such as Johann Bernoulli and Christian Goldbach, with whom he corresponded extensively on number theory, but the result that first caught the attention of the wider mathematics community was his 1734 evaluation of the sum of reciprocals of squares, dubbed the Basel problem in honor of his birth town (Calinger, 2007). The unexpected total of $\frac{\pi^2}{6}$ stems from a creative factorization of the sine function as if it were a polynomial with infinitely many roots, a solution that is characteristic of Euler's elegant manipulations.

In 1727, Euler joined the faculty of the Saint Petersburg Academy of Sciences, which was part of Peter the Great’s effort to modernize the Russian Empire and make it a hotspot of academic activity in Europe. The emperor had only established the university a few years earlier, in 1724, in consultation with Gottfried Wilhelm Leibniz, who was a supporter of international collaboration. Euler has been credited as a source of the institution’s rapid, although short-lived, flourishing (Schulze, 1985). Indeed, he contributed significantly to the journal *Commentārīī Acadēmiae Scientiārum Imperiālis Petropolitānae* ever since its second volume in 1729. Volume 8 alone included 11 of his articles, and as a result, its table of contents is full of the word *Eiusdem* used to denote repeated authorship (Euler, 1741). An example of one of these 11 papers, excerpted below, is his “Solūtiō Problēmatis ad Geōmetriam Sitūs Pertinentis” (E53), which set out to solve a problem which Euler was also discussing in letters with the mayor of Danzig, Carl Gottlieb Ehler, around the time of the writing of this paper: Is it possible to journey across all seven bridges of Königsberg exactly once in succession? Euler realized that he could generalize his technique to all possible systems of bridges, stating that what is now called an Eulerian path exists if and only if there are exactly zero or two regions that have an odd number of bridges leading into them. Although he may have not yet employed any of the associated terminology, his paper is considered the origin of graph theory, as the islands, when shrunken down to points, become vertices and the bridges connecting them become edges (see Figure 1). A related puzzle that can be systematically solved — or revealed to be unsolvable — through this groundbreaking result in graph theory is whether a certain line drawing is traceable by one continuous stroke (Ball, 1905): Every intersection in the given diagram is analogous to a

vertex, and the resulting line segments, all of which must be traced, are equivalent to edges (see Figure 2).

In the century following Euler's publication, graph theory was applied to a wide variety of problems, including whether a graph possesses a Hamiltonian circuit, the more difficult vertex-based counterpart to the Eulerian path that must pass through each vertex exactly once. Euler's formula $V - E + F = 2$ for the number of vertices, edges, and faces in convex polyhedra was, as its name suggests, also first discovered in 1752 by Euler, whose original proof relied on meticulous but faulty and inexhaustive casework. It was later translated in terms of planar (i.e. non-intersecting) graphs by Augustin-Louis Cauchy in 1813 through a projection of the vertices and edges of the solid onto the basic objects with the same names in a graph. Finally, the four-color theorem, which initially pertained to the minimum number of colors needed in a map such that adjacent countries differ in color, can be naturally converted into a graph problem concerning the coloring of vertices by representing adjacency with an edge. It has notoriously evaded an elegant proof, and indeed the conjecture that a planar graph requires only four colors at most was only shown to be true in 1976, with the help of a computer thoroughly checking the 1482 fundamental cases of graphs. Graph theory is in fact very prominent in computer science generally, with numerous search or optimization algorithms, including Hierholzer's described below, under its domain (Wilson, 2013).

Euler's solution to the Königsberg problem is often also categorized as an early work in topology due to its deviation from what was long accepted as the main object of mathematics, a field encapsulated

by Aristotle as a science of quantity: "[T]he mathematician . . . strips off all the sensible qualities, e.g. weight and lightness, hardness and its contrary, and also heat and cold and other sensible contrarieties, and leaves only the quantitative and continuous" (Metaphysics, 11.3, 1061a29-33).[1] Indeed, in contrast to natural philosophy and physics, which at the time of the Ancient Greeks were defined by logical syllogisms and a qualitative observational approach, respectively, math and its two subfields of arithmetic and geometry were distinguished by a focus on magnitudes and other measurable concepts that usually, but not always, exist in the real world (Anderson, 1972). This perspective seems to have survived even past the discoveries of mathematical objects that, at first glance, do not correspond to any element of reality, such as the imaginary numbers, which were first analyzed with a computational purpose in mind, namely to allow the cubic formula to output a real root even if it were to involve square roots of negative numbers. Euler's Königsberg problem was hence quite novel at the time because, as he stresses in the article's opening paragraph, it "neither requir[es] the determination of quantities, nor admit[s] a solution of calculation through the help of quantities" (*neque dēterminātiōnem quantitātum requīreret, neque solūtiōnem calculī quantitātum ope admitteret*). Instead, all the information in a graph is encoded in the edges, a relation between two vertices that, in and of itself, possesses no quantitative properties (except in the case of so-called weighted graphs, which may be beneficial in computer science applications). Graph theory, therefore, was one of the first fields to transform mathematics into what may be termed a science of relations. Topology, which imitates the term *geōmetria sitūs* through its Greek roots τόπος, "position," and λόγος, "study," later emerged in the 19th century as a formalization of the connectivity of points

[1] This translation is taken from Anderson (1972).

or sections of shapes and provided a medium in which to study holes or equivalences between, say, a coffee mug and a donut. And, at least according to modern-day string theorists, topology bridges — pardon the pun — the gap between general relativity and quantum mechanics, thereby making science in general a science of relations at its core.

Source Text: Our Latin text comes from Euler (1741).

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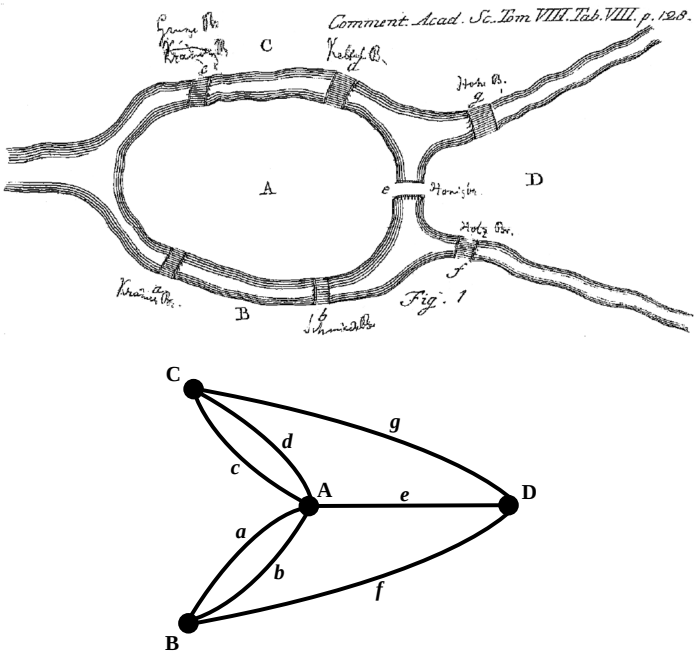


Figure 1: Euler's original Königsberg diagram (above) can be converted into a graph with vertices and edges (below). Image credit: Wikimedia Commons, public domain.

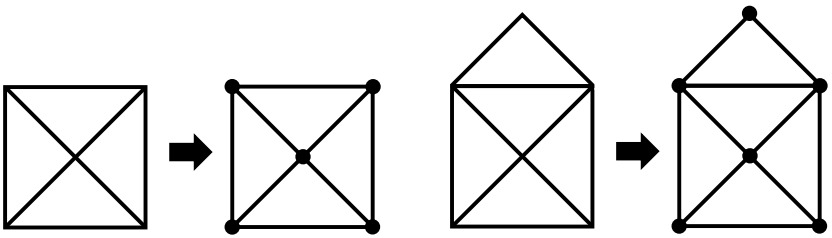


Figure 2: On the left is a classic example of a line drawing that cannot be traced without lifting a pen or retracing a part of the design — the corresponding graph violates Euler's conditions for the existence of an Eulerian path. An Eulerian path does exist in the modified diagram on the right, known in Germany as "das Haus vom Nikolaus" ("Santa's House").

Solūtiō Problēmatis ad Geōmetriam Sitūs Pertinentis

1. Praeter illam Geōmetriae partem, quae circā quantitātēs versātur, et omnī tempore summō studiō est exculta, alterīus partis etiamnum admodum ignōtae prīmus mentiōnem fēcit *Leibnitzius*, quam Geōmetriam sitūs vocāvit.

---

solūtiō, -ōnis f.: solution

problēma, -atis n.: problem (from the Greek πρόβλημα, something "put forward")

geōmetria, -ae f.: geometry (from the Greek γεωμετρία, "earth-measuring")

situs, -ūs m.: position

pertineō, -ēre, -uī, – (+ *ad*): pertain to, belong to

praeter (prep. + acc.): (here) aside from

quantitās, -ātis f.: amount; quantity; magnitude

versor, -ārī, -ātus sum (+ *circā* and acc.): be occupied with, be concerned with (from *versō (1)*, "to turn much")

studium, -iī n.: zeal, eagerness

excolō, -ere, -uī, -cultus: work carefully; improve, perfect

etiamnum (adv.): yet, still, even now; until then

admodum (adv.): completely; quite

ignōtus, -a, -um: unknown

mentiō, -ōnis f.: mention

Leibnitzius, -iī m.: Godefridus Guilielmus Leibnitzius (the Latinized version of Leibniz's name; see below)

---

omnī tempore: *all the time*; ablative of time when

summō studiō: ablative of manner

alterīus partis . . . ignōtae prīmus mentiōnem fēcit *Leibnitzius*: *alterīus partis . . . ignōtae* is an objective genitive governed by *mentiōnem*; *prīmus* modifies *Leibnitzius* but functions as a quasi-adverb; Leibniz's "geometry of position" was an attempt to rework the entirety of geometry purely in terms of distances (like in the modern study of metric spaces), and unlike his contemporaries, he emphasized the importance of the space in which points lie; Euler's Königsberg problem has little to do with distance, but, as done here, Leibniz's *geōmetria sitūs* was frequently cited as inspiration for other, unrelated reconstructions of geometry (e.g. Henri Poincaré's seminal text on topology, *Analysis Sitūs*) (De Risi, 2013)

[. . .] Quamobrem, cum nūper problēmatis cuiusdam mentiō esset facta, quod quidem ad geōmetriam pertinēre vidēbātur, at ita erat comparātum, ut neque dēterminātiōnem quantitātum requīreret, neque solūtiōnem calculī quantitātum ope admitteret, id ad geōmetriam sitūs referre haud dubitāvī, ⇒

---

quamobrem: for which reason, therefore
nūper (adv.): recently
comparō (1): (here) prepare, arrange, establish
dēterminātiō, -ōnis f.: determination
requīrō, -ere, -quīsīvī, -quīsītus: seek; ask, demand; require
calculus, -ī m.: calculation (originally the diminutive of *calx, calcis f.*, “stone,” because little stones were used for counting boards)
ops, opis f.: help
admittō, -ere, -mīsī, -missus: allow, admit
referō, -ferre, rettulī, relātus: (here, with *ad*) relate to, connect to
haud (adv.): not at all
dubitō (1): doubt

---

cum . . . mentiō esset facta: circumstantial *cum* clause with a pluperfect subjunctive verb

at ita erat comparātum: this clause belongs to the relative clause; *ita* anticipates the result clause introduced by *ut* and containing the imperfect subjunctive verbs *requīreret* and *admitteret*

quantitātum ope: *through the help of quantities*; ablative of means

(continued) praesertim quod in eius solūtiōne sōlus situs in cōnsīderātiōnem veniat, calculus vērō nūllīus prōrsus sit ūsūs. Methodum ergō meam quam ad huius generis problēmata solvenda invēnī, tanquam specimen Geōmetriae sitūs hīc expōnere cōnstituī.

---

praesertim (adv.): especially, particularly
cōnsīderātiō, -ōnis f.: consideration
vērō (adv.): in truth; but, and (postpositive)
prōrsus (adv.): (here, intensifying a negative) at all
ūsus, -ūs m.: use
methodus, -ī f.: method (from the feminine Greek μέθοδος)
solvō, -ere, -ī, solūtus: (here) solve
inveniō, -īre, -vēnī, -ventus: find
tanquam = tamquam (adv.): as much as; like, as
specimen, speciminis n.: model, example (cf. specimen)
expōnō, -ere, -posuī, -positus: (here) set forth, explain
cōnstituō, -ere, -ī, -ūtus: (here) resolve, decide

---

quod: it is not a relative pronoun and instead means “because”

nūllīus prōrsus sit ūsūs: *is of no use at all*; genitive of description

quam ad huius generis problēmata solvenda invēnī: *which I found to solve problems of this kind*; relative clause with antecedent *methodum*; this gerundive phrase is equivalent to the gerund *ad huius generis problēmata solvendum*; a gerundive phrase with *ad* expresses purpose

2. Problēma autem hoc, quod mihi satis nōtum esse perhibēbātur, erat sequēns: Rēgiomontī in Borūssiā esse īnsulam A Der Kneiphof dictam, fluviumque eam cingentem in duōs dīvidī rāmōs, quemadmodum ex figūrā vidēre licet; rāmōs vērō huius fluviī septem īnstrūctōs esse pontibus, *a*, *b*, *c*, *d*, *e*, *f*, et *g*.

---

satis: (as adv.) sufficiently, enough

nōtus, -a, -um: known, famous (from *nōscō, -ere, nōvī, nōtus*, “to learn”)

perhibeō, -ēre, -uī, -itus: (here) say, assert

Rēgiomontum, -ī n.: Königsberg (from the literal translation *Rēgius Mōns*, “king’s mountain”; its modern counterpart is the city of Kaliningrad in Russia)

Borūssia, -ae f.: Prussia

Der Kneiphof: German name for the central island of Königsberg

fluvius, -iī m.: river

cingō, -ere, cīnxī, cīnctus: go around, surround

dīvidō, -ere, -vīsī, -vīsus: divide

rāmus, -ī m.: branch; arm (of a river)

quemadmodum: in which manner; just as

figūra, -ae f.: figure

licet, -ēre, -uit, – (+ inf.): (impers.) it is allowed, it is possible

septem (indecl.): seven

īnstrūctus, -a, -um: furnished, provided (from *īnstruō, -ere, -strūxī, -strūctus*)

pōns, pontis m.: bridge

---

erat sequēns: *was the following*

Rēgiomontī: locative form

esse īnsulam A . . . fluviumque eam cingentem . . . dīvidī: two indirect statements with subjects *īnsulam A* and *fluvium* and infinitives *esse* and *dīvidī*, respectively; these indirect statements are in apposition to *problēma*; the *eam* refers back to *īnsulam* and is the object of the PAP *cingentem*, which modifies *fluvium*; see Figure 3 on page 213 for the corresponding diagram

rāmōs . . . īnstrūctōs esse: a continuation of the above set of indirect statements

Circā hōs pontēs iam ista prōpōnēbātur quaestiō: num quis cursum ita īnstituere queat, ut per singulōs pontēs semel et nōn plūs quam semel trānseat. Hocque fierī posse, mihi dictum est, aliōs negāre aliōs dubitāre; nēminem vērō affirmāre.

---

circā (prep. + acc.): (here) about, concerning
prōpōnō, -ere, -posuī, -positus: set; give, propose
quaestiō, -ōnis f.: inquiry, question
num: (here) whether
cursus, -ūs m.: course
īnstituō, -ere, -ī, -ūtus: (here) undertake
queō, -īre, -īvī, -itus: be able to
per (prep. + acc.): (here) over
singulī, -ae, -a: individual
semel (adv.): once
plūs, plūris: more (comparative of *multus, -a, -um*; in the singular, usually only neuter)
trānseō, -īre, -iī, -itus: go over, cross
negō (1): deny
nēmō, -inis m./f.: no one, nobody
affirmō (1): assert, affirm; confirm

---

num quis cursum ita īnstituere queat: in a deliberative question, the subjunctive verb (*queat*) remains present even if the main verb (*prōpōnēbātur*) is imperfect; a *quis* that follows *num* functions like *aliquis*; *ita* anticipates the result clause introduced by *ut*

semel et nōn plūs quam semel: *once and not more than once*; the comparison uses *quam*

Hocque fierī posse: indirect statement set up by *negāre*, *dubitāre*, and *affirmāre*

aliōs negāre aliōs dubitāre: indirect statement introduced by *dictum est*; *aliōs . . . aliōs* should be translated as "some . . . others"

Ego ex hōc mihi sequēns maximē generāle fōrmāvī problēma: quaecunque sit fluviī figūra et distribūtiō in rāmōs, atque quīcunque fuerit numerus pontium, invenīre utrum per singulōs pontēs semel tantum trānsīrī queat, an vērō secus?

---

maximē (adv.): (here, with adj.) very
generālis, -e: general
fōrmō (1): form, conceive
quīcumque, quaecumque, quodcumque (-cunque): whoever, whatever; every, each
distribūtiō, -ōnis f.: division
numerus, -ī m.: number
utrum . . . an: whether . . . or
tantum: only
secus (adv.): (in the second of two deliberative clauses) not

---

invenīre: *to find*; infinitive in a problem statement, grammatically in apposition to *problēma*

utrum . . . an vērō secus: *whether . . . or in truth not*; indirect question with a subjunctive verb *queat*

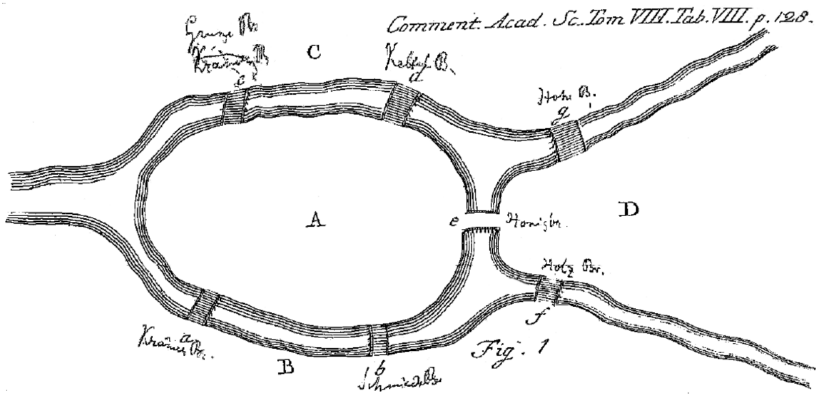


Figure 3: Der Kneiphof, as illustrated in a diagram from the paper. In this annotated version of the paper accessible on *The Euler Archive* (`https://scholarlycommons.pacific.edu/euler-works/53/`), the names of the bridges are also provided — they are, in counterclockwise order, the Köttel Brücke, Grüne Brücke, Krämer Brücke, Schmiede Brücke, Holz Brücke, Honigbrücke, and Hohe Brücke. Image credit: Wikimedia Commons, public domain.

9. In cāsū igitur pontium trānseundōrum Rēgiomontānō, quia in īnsulam A quīnque pontēs dēdūcunt *a*,*b*,*c*,*d*,*e*, necesse est ut in dēsignātiōne trānsitūs per hōs pontēs littera A ter occurrat.

---

cāsus, -ūs m.: case
igitur: therefore
Rēgiomontānus, -a, -um: pertaining to Königsberg, Königsbergian
dēdūcō, -ere, -dūxī, -ductus: lead
necesse est: it is necessary that
dēsignātiō, -ōnis f.: marking out, describing, designation
trānsitus, -ūs m.: a going over, passage
littera, -ae f.: letter
ter (adv.): thrice, three times
occurrō, -ere, -ī, -cursus: (here) appear, occur

In cāsū igitur pontium trānseundōrum Rēgiomontānō: notice the bracketing of the gerundive phrase *pontium trānseundōrum*, “of crossing (the) bridges,” between the noun-adjective pair *cāsū . . . Rēgiomontānō*

necesse est ut . . . occurrat: *necesse est* is usually accompanied by an infinitive clause, but in this case the predicate of *est* is a noun clause introduced by *ut*

Deinde littera B, quia in regiōnem B trēs pontēs condūcunt, bis dēbet occurrere, similīque modō littera D bis dēbet occurrere, atque etiam littera C bis. In seriē ergō octo litterārum, quibus trānsitus per septem pontēs dēbēret dēsignārī, littera A ter adesse dēbēret, litterārum vērō B, C, et D ūnaquaeque bis;

---

regiō, -ōnis f.: region
condūcō, -ere, -dūxī, -ductus: lead together
bis (adv.): twice, two times
similis, -e: similar
seriēs, -ēī f.: series
octo (indecl.): eight
dēsignō (1): designate, represent
adsum, -esse, -fuī, -futūrus: be present
ūnusquisque, ūnaquaeque, ūnumquidque: each one, each

---

quibus . . . dēbēret dēsignārī: a relative clause of characteristic with antecedent *octo litterārum* and the subjunctive verb *dēbēret*; *quibus* is an ablative of means

dēbēret: *should have had to*; a potential subjunctive

(continued) id quod in seriē octo litterārum omnīnō fierī nequit. Ex quō perspicuum est, per septem pontēs Rēgiomontānōs tālem trānsitum īnstituī nōn posse.

---

omnīnō (adv.): altogether, at all
nequeō, -īre, -īvī, -itus: cannot
perspicuus, -a, -um: clear, evident
tālis, -e: such

---

id quod . . . fierī nequit: *which cannot happen . . .* ; *id* is the antecedent of *quod* and refers to the entire previous sentence; Euler's proof goes as follows: we first create a sequence of letters representing the regions traveled to (e.g. B → A → C → ⋯ for a path starting at B, continuing to A, then C, and so on); in Königsberg (see Figure 4 below), there exist 7 bridges or arrows between the letters, of which there must therefore be 8 by fenceposting; if a region has n bridges connected to it, its letter must appear $\lfloor \frac{n+1}{2} \rfloor$ times (i.e. $\frac{n}{2}$ times when n is even and $\frac{n+1}{2}$ times when n is odd) because, for every pair of consecutive bridges, one of the two enters the region and the next exits it (e.g. → A →); hence, the sequence of 8 letters consists of $3 + 2 + 2 + 2 = 9$ letters, a clear contradiction

Ex quō: the relative pronoun (whose antecedent is the previous sentence) can be translated as a demonstrative

per septem pontēs . . . īnstituī nōn posse: indirect statement introduced by *perspicuum est*

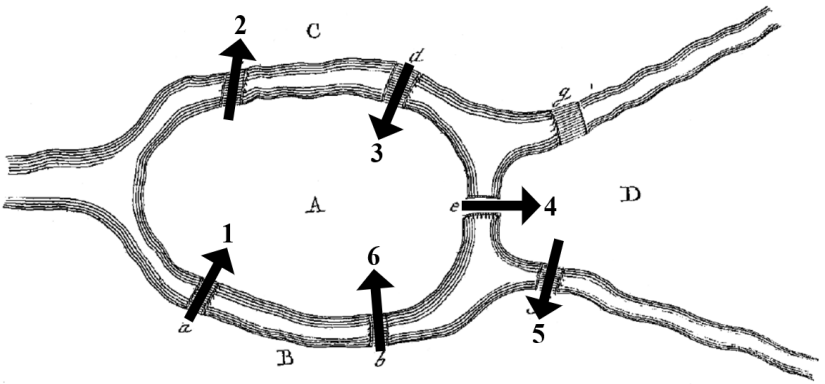


Figure 4: A failed Eulerian path with the sequence B → A → C → A → D → B → A. Image credit: modification of image from Wikimedia Commons, public domain.

20. Cāsū ergō quōcunque prōpositō statim facillimē poterit cognōscī utrum trānsitus per omnēs pontēs semel īnstituī queat an nōn, ope huius rēgulae. Sī fuerint plūrēs duābus regiōnēs, ad quās dūcentium pontium numerus est impār, tum certō affirmārī potest tālem trānsitum nōn darī.

---

statim (adv.): at once
cognōscō, -ere, -nōvī, -nitus: gain knowledge, understand, perceive
rēgula, -ae f.: rule (cf. regular)
impār, imparis: unequal; (here) odd
certō (adv.): surely, with certainty

Cāsū . . . quōcunque prōpositō: *in each given case*; *prōpositō* is a PPP modifying *cāsū*, which, in the ablative, can mean “in (a) case” even without a preposition

utrum . . . an nōn: *whether . . . or not*; indirect question with the subjunctive verb *queat*

ope huius rēgulae: *through the help of this rule*; ablative of means

Sī fuerint plūrēs duābus regiōnēs: *fuerint* is a future perfect indicative in a conditional (which is usually translated in the present in English); *duābus* is an ablative of comparison

ad quās dūcentium pontium numerus est impār: the antecedent of *quās* is *regiōnēs*; *dūcentium* is a PAP modifying *pontium*; remember that, if a region has n bridges, then the region or corresponding letter appears $\frac{n}{2}$ times in the travel sequence if n is even and $\frac{n+1}{2}$ times if n is odd; since every bridge connects to exactly two regions, the total number of letters in the sequence is equal to the number of bridges plus $\frac{N_{\text{odd}}}{2}$, where N_{odd} is the number of regions with an odd number of outgoing bridges; since the number of letters total must be equal to 1 plus the number of bridges (as determined above), N_{odd} cannot be greater than 2 if an Eulerian path exists; additionally, N_{odd} cannot be odd as the number of letters must be an integer

Sī autem ad duās tantum regiōnēs dūcentium pontium numerus est impār, tunc trānsitus fierī poterit, sī modo cursus in alterā hārum regiōnum incipiātur.

---

sī modo: if only; provided that **incipiō, -ere, -cēpī, -ceptus:** begin

---

ad duās tantum regiōnēs dūcentium pontium numerus: *the number of bridges leading to only two regions*

in alterā hārum regiōnum: *in either one of these (two) regions*; we now only increase the aforementioned lower bound by $\frac{1}{2}$ for exactly two of the regions, implying that the total number of letters is 1 plus the number of bridges, as required; also, the Eulerian path must begin in one of the regions with an odd number of bridges and must end in the other (because, for such a region, we cannot create entrance-exit pairs containing all bridges)

incipiātur: a present subjunctive in a proviso clause

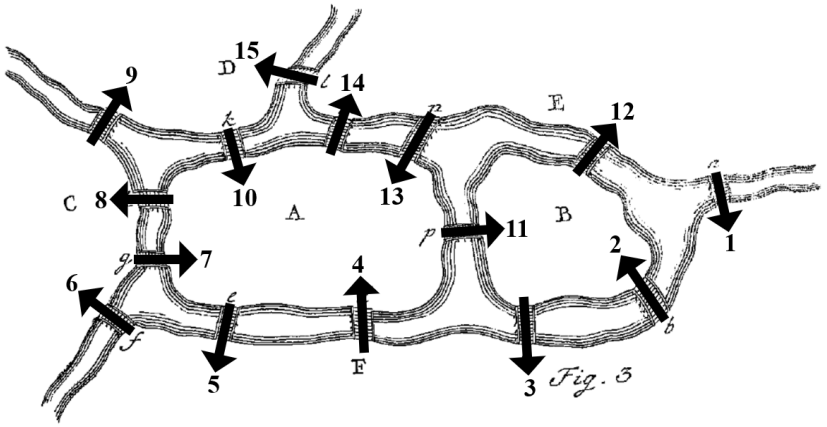


Figure 5: An Eulerian path with the sequence E → F → B → F → A → F → C → A → C → D → A → B → E → A → E → D. Image credit: modification of image from Wikimedia Commons, public domain.

Sī dēnique nūlla omnīnō fuerit regiō, ad quam pontēs numerō imparēs condūcant, tum trānsitus dēsīderātō modō īnstituī poterit, in quācunque regiōne ambulandī initium pōnātur. Hāc igitur datā rēgulā problēmatī prōpositō plēnissimē satisfit.

---

dēnique (adv.): finally
dēsīderō (1): long for, desire
initium, -iī n.: beginning
datus, -a, -um: given
plēnē (adv.): fully, thoroughly, perfectly
satisfaciō, -ere, -fēcī, -factus (+ dat.): give satisfaction to

---

ad quam pontēs numerō imparēs condūcant: the verb is subjunctive due to the counterfactuality of the statement (there are no regions with an odd number of bridges); notice again the bracketing of the ablative of respect *numerō*, "in number," between the noun-adjective pair *pontēs . . . imparēs*

dēsīderātō modō: *in the desired manner*, i.e. with each bridge being crossed only once; while the number of letters total in this case seems to be exactly the number of bridges, and not one more as we'd expect, it is important to recognize that the starting region appears one extra time, as it is repeated at both the beginning and end of the sequence; note that the Eulerian path can now start anywhere and is now an Eulerian tour because it starts and ends in the same region

Hāc . . . datā rēgulā: ablative of means

problēmatī prōpositō . . . satisfit: literally *satisfaction is given to the proposed problem*, but more idiomatically *the proposed problem is satisfactorily solved*; the passive forms of *satisfaciō* are constructed from the irregular passive forms of *faciō*

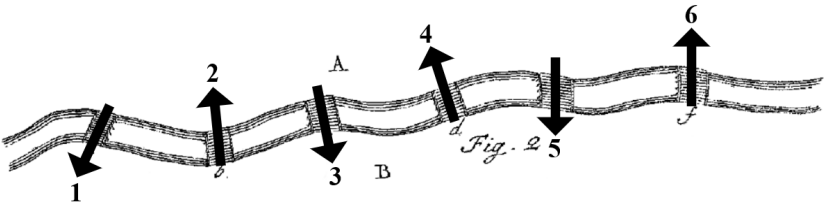


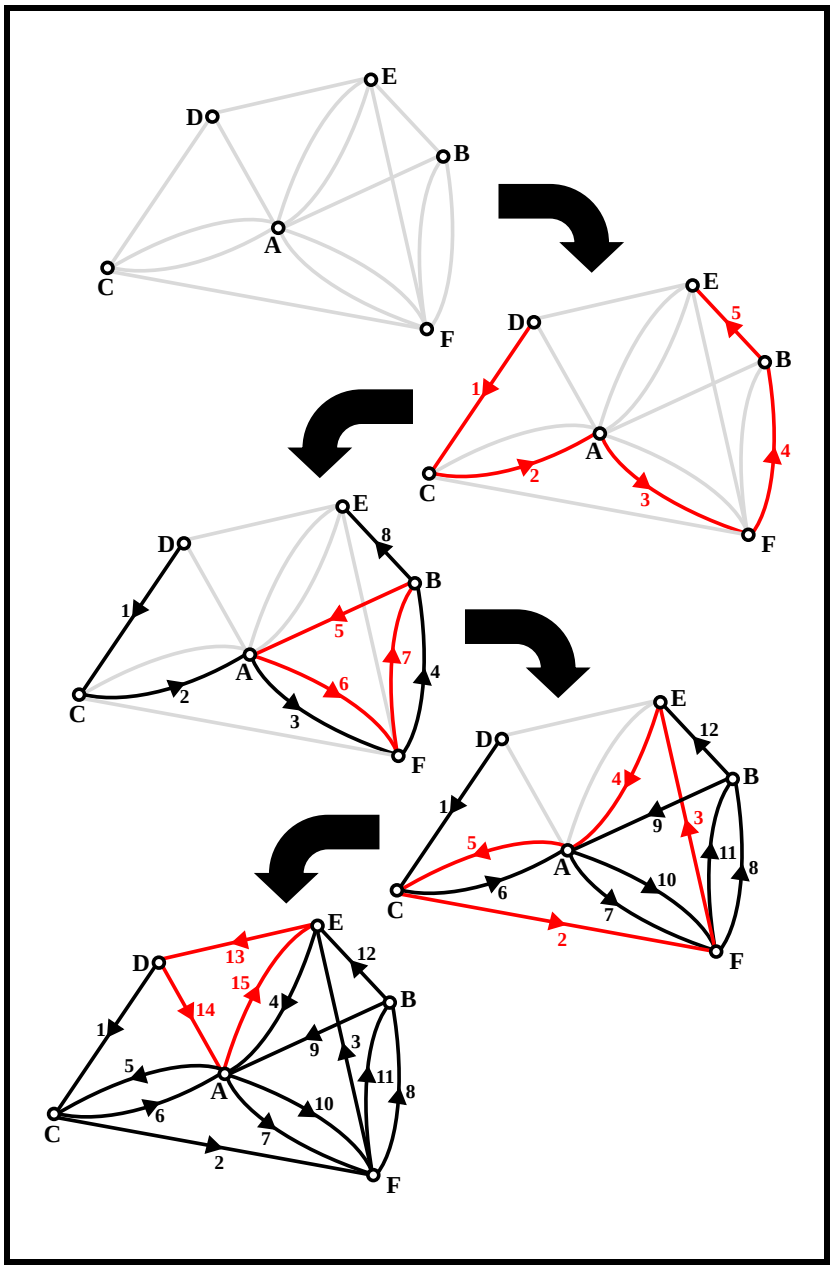
Figure 6: An Eulerian path with the sequence A → B → A → B → A → B → A. Image credit: modification of image from Wikimedia Commons, public domain.

An Existence Proof for Eulerian Paths

Figure 7 (facing page): Although Euler does correctly prove that an Eulerian path exists only if zero or two regions have an odd number of bridges, he only briefly argues in support of the converse at the end of the paper, where he states that the trivial elimination of a pair of bridges connecting the same two regions can (minimally) simplify the task of finding the Eulerian path.

In a paper posthumously published in 1873, Carl Hierholzer outlined the first actual algorithm finding an Eulerian path for any graph satisfying Euler's condition (Hierholzer, 1873): Consider such a graph where the number of vertices with an odd number of outgoing edges is either zero or two. First, find some path passing through all vertices with the starting and ending vertices indicated in Euler's article. In the case that these two vertices are identical, this initial path is a cycle. If a certain vertex v has a nonzero (but necessarily even) number of remaining outgoing edges, construct a cycle that travels from and then returns to v and then attach it to the preexisting path. Specifically, after the path enters v, it can be continued by traversing the cycle before resuming the rest of the path.

This process reduces the number of outgoing edges not currently included in the path by two, thereby guaranteeing that the path eventually encompasses all edges and becomes Eulerian. Such a cycle must exist because whenever a vertex is entered, it can be left by passing through another edge (all vertices have an even number of edges), which once again reduces the number of untraveled edges. Unless the partial cycle happens to return to v earlier, eventually the only edge left would be the one that returns to v, which, once added, completes the cycle. See a visualization of Hierholzer's algorithm below with the graph corresponding to Figure 5 — the v's considered in each step are B, C, and E.


E
D
B
A
C
F
E
5
D
B
1
A
4
2
C
3
F
E
8
D
B
1
5
7
A
4
6
C
2
3
F
E
12
D
B
1
4
9
3
5
A
11
8
10
6
C
7
2
F
E
13
12
D
15
14
4
B
1
9
3
5
A
11
8
10
6
C
7
2
F

Chapter 9 — Gauss: *Disquīsītiōnēs Arithmēticae*

The German mathematician, physicist, and astronomer Carl Friedrich Gauss (1777 CE – 1855 CE) was by all accounts a prodigy, as is often highlighted through a legend depicting the ten-year-old student's independent rederivation[1] of the formula $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. While today's popular math enrichment programs may make this childhood accomplishment seem less impressive to a modern reader, this genius from a working-class background attracted the attention — and the funding — of the Duke of Brunswick. At 22, Gauss received his PhD at the University of Helmstedt for a dissertation on the Fundamental Theorem of Algebra (FTA), which states that every polynomial can be decomposed into only linear and quadratic factors, which have real and complex roots, respectively (Dunnington, 1955). Gauss first pointed out circular reasoning or dubious limit arguments implicit in earlier proofs of this statement before sharing his own attempt (Baltus, 2004). Although Gauss's proof, like its predecessors, is not without substantial leaps in reasoning, its basic argument is so easily mendable that many modern historians credit the young scholar for the first sufficiently rigorous proof of the FTA (Basu and Velleman, 2017). Gauss would revisit the theorem three times throughout his life.

Only a couple of years later, in 1801, Gauss successfully predicted the position of Ceres using only the limited data collected by astronomers before the (dwarf) planet was lost behind the sun. The method of least squares used for this calculation marks Gauss's entrance into the field of statistics, where he is arguably most famous

[1] After duplicating each term in the sum and thereby doubling the total, pair each term k with $n + 1 - k$ to obtain n copies of $n + 1$.

for his treatment of the normal or, as it is now also called, Gaussian distribution. In one of the classic priority disputes in mathematics, Gauss alleged without evidence that he had devised the method of least squares in 1794, even though he only published it formally in 1809. As it happens, another mathematician, Adrien-Marie Legendre, whose 1805 work discussing the method Gauss himself referenced, felt he deserved primary credit for the discovery. Part of his case was that so many mathematical theorems had already been attributed to Gauss (Dutka, 1995)![2]

By far the most substantial of Gauss's early undertakings was his first book, the *Disquīsītiōnēs Arithmēticae*, published in 1801 after many delays and with a difficult release due to chaos brought on in Europe by the French Revolution (Bullynck, 2025). The work's area of focus is number theory, which Gauss helped establish as a field in its own right, at a time when it was simply classified as a subdiscipline of arithmetic. The *Disquīsītiōnēs* is divided into three main units that correspond to different stages of its writing process: The first three sections connect the findings of various mathematicians, from the Ancient Greeks to his contemporaries, within the general framework of modular arithmetic. As Gauss explains in the preface partially excerpted below, he accidentally rediscovered many known results in number theory back in 1795 due to a lack of easy access to contemporary research in his early studies (Dunnington, 1955). In

[2]Legendre's original French letter is incorporated into the first part of the tenth volume of the *Werke*, a collection of Gauss's writing: "En Mathématiques il arrive très souvent qu'on trouve les mêmes choses qui ont été trouvées par d'autres et qui sont bien connues; c'est ce qui m'est arrivé nombre de fois, mais je n'en ai point fait mention et je n'ai jamais appellé *prīncipium nostrum* un pr[incipe] qu'un autre avait publié avant moi. Vous êtes assez riche de [votre] fonds, Monsieur, pour n'avoir rien à envier à personne." ("In Mathematics, it very often happens that one finds the same things that have been found by others and that are well-known; that is what has happened to me many times, but I have not made mention of it and have never called a pr[inciple] that someone else had published before me a *prīncipium nostrum*. You are rich enough on [your] own, Sir, to have nothing to envy anyone for.")

April of 1796, he proved the quadratic reciprocity law, which Gauss characterized as the *theōrēma aureum* in his diary and as the *theōrēma fundāmentāle* in the *Disquīsītiōnēs* itself, foreshadowing its role as the foundation of modern algebraic number theory. One of the few to truly appreciate and engage in this number-theoretic material at the time was Sophie Germain, a mathematician and frequent correspondent of Gauss who would go on to become an influential figure in the centuries-long legacy of Fermat's Last Theorem (Neumann, 2005). Indeed, Gauss received the most immediate recognition for his novel Euclidean construction of the regular heptadecagon (a.k.a. 17-gon) using only a straightedge and compass, which is the focus of the book's last unit. Perhaps in imitation of the cylinder-sphere design supposedly engraved on Archimedes's tomb (see Cicero's excerpt), Gauss requested that his achievement be literally carved in stone on his grave, but the heptadecagon's visual similarity to a circle is said to have made this impossible (Dunnington, 1955). This geometric result may have attracted the most attention from an audience raised in Euclidean geometry, but even the earliest reviewers of the *Disquīsītiōnēs* could not ignore its comprehensiveness, the clarity of the writing, and the precision of the notation (Bullynck, 2025).

Even within the first three sections of the *Disquīsītiōnēs* that mostly summarize the works of earlier number theorists, Gauss is original and exemplary in his formalism, for instance inventing the now standard symbol $\equiv$ for congruence. In the second excerpt below, he defines a to be congruent to b modulo m (i.e. $a \equiv b \mod m$) if and only if $a - b$ is divisible by m, and he proves that all integers uniquely correspond to remainder or "residue" modulo m ranging from 0 to $m - 1$. A nice way to visualize so-called modular arithmetic is with a clock (see Figure 1 below), where we start at 0,

then move to 1, and continue until we reach $m - 1$, at which point we return to 0. Congruence in Gauss's modular arithmetic is the first explicit example of an equivalence relation, a generalization of the notion of equality, even if the term "equivalence relation" was only introduced over a century later. Whereas equality can only equate an object with itself, an equivalence relation allows multiple objects to be considered equivalent to each other under certain basic conditions, effectively collapsing them into a representative object known as an equivalence class. In modular arithmetic, 1, 13, and any integer of the form $1 + 12n$ can be treated as identical to the number 1 modulo 12, with each residue modulo 12 being its own equivalence class. Another example of an equivalence relation, from Euclidean geometry, is similarity, which equates all triangles with the same proportions without regard for their position, orientation, or scaling. Equivalence classes are a recurring theme of the *Disquīsītiōnēs*, notably reemerging in the context of binary quadratic forms (Dahan-Dalmedico and Peiffer, 1986).

Gauss envisioned modular arithmetic to have "connect[ions] to greater things" (*cum maiōribus connexam*), and it is worth mentioning some of them. As an early practical application, Gauss employed modular arithmetic to develop a simple algorithm calculating the date of Easter in the Gregorian calendar for every year up to 4199. Gauss allegedly used his algorithm to identify his own birthdate of April 30$^{\text{th}}$, since his mother could only remember that he had been born eight days before Ascension Day and, therefore, 31 days after Easter (Teets, 2019)! On the more theoretical side, the set of residues modulo m under addition, the classic example of a finite group, is embedded into the structure of the solutions to the polynomial $x^m - 1 = 0$, which are collectively known as the mth

roots of unity and can be represented as equally-spaced points along the unit circle in the complex plane (Dahan-Dalmedico and Peiffer, 1986). This appearance of modular arithmetic in geometry is precisely the conceptual link between Gauss's construction of the heptadecagon (which requires the division of a circle into 17 arcs of equal length) and the rest of the *Disquīsītiōnēs*. Perhaps surprisingly, present-day cryptographic security protocols such as RSA or Diffie-Hellman heavily depend on modular arithmetic as well, exploiting the fact that congruence can keep the operations used to encode a secret message hidden from eavesdroppers.

Source Text: Our Latin text comes from Gauss (1801).

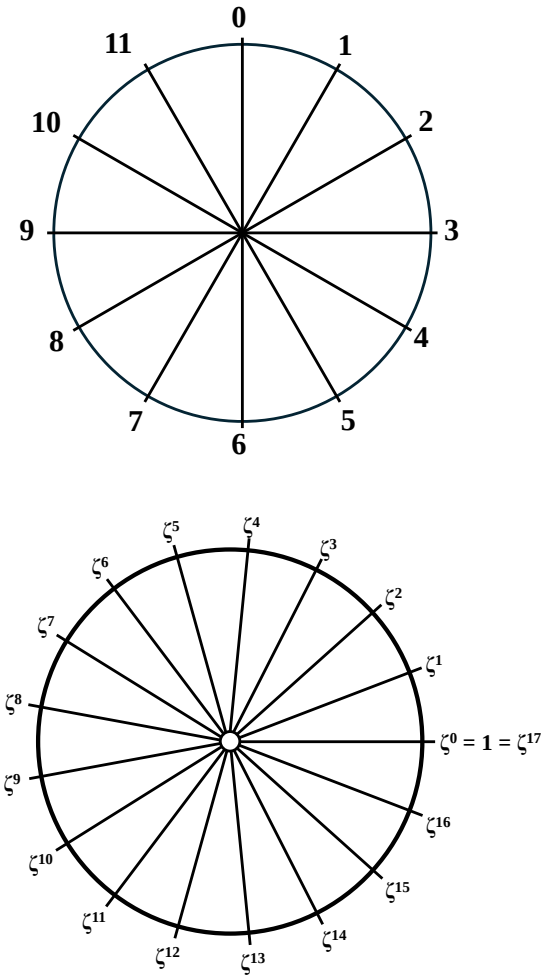


Figure 1: The upper circle is a representation of the residues modulo 12, while the lower image depicts the 17^{th} roots of unity arranged on the unit circle in the complex plane. This visualization makes the relationship between the two concepts clear: If the principal n^{th} root of unity is labeled ζ, then ζ^m under multiplication is analogous to m modulo n due to the fact that $\zeta^n = 1$ by definition. Indeed, for any m, ζ^m can be reduced to $\zeta^{r+kn} = \zeta^r \cdot (\zeta^n)^k = \zeta^r \cdot 1^k = \zeta^r$, where r is the residue of m modulo n.

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Figure 2: In 1990, the German Central Bank dedicated the 10-mark banknote in its BBk III series to Gauss, the same series that included a banknote for Merian (see page 162). The obverse has Göttingen in the backdrop behind a Gaussian distribution, while the reverse honors the *heliotrope*, a device invented by Gauss for long-distance measurements in surveying. Image credit: Wikimedia Commons, work by YavarPS, licensed under CC BY-SA 4.0 (`https://creativecommons.org/licenses/by-sa/4.0/`).

Disquīsītiōnēs Arithmēticae

Praefātiō

Disquīsītiōnēs in hōc opere contentae ad eam Mathēseōs partem pertinent, quae circā numerōs integrōs versātur, frāctīs plērumque, surdīs semper exclūsīs.

---

disquīsītiō, -ōnis f.: inquiry, investigation

arithmēticus, -a, -um: arithmetic

praefātiō, -ōnis f.: preface

opus, operis n.: work

contineō, -ēre, -uī, -tentus: (here) contain

mathēsis, -eōs f.: mathematics (from the Greek μάθησις, which has the same root as "mathematics," namely μανθάνειν, "to learn")

pertineō, -ēre, -uī, – (+ *ad*): pertain to, belong to

numerus, -ī m.: number

integer, -gra, -grum: (here, in a mathematical context) integral

versor, -ārī, -ātus sum (+ *circā* and acc.): occupied with, be concerned with (from *versō (1)*, "to turn much")

frāctus, -a, -um: fractional (lit. "broken")

plērumque (adv.): for the most part, mostly

surdus, -a, -um: deaf; unwilling to listen; (in math) irrational, surd

exclūdō, -ere, -clūsī, -clūsus: shut out, exclude

---

ad eam Mathēseōs partem: *eam* functions as a demonstrative adjective modifying *partem* and is translated as "this"; notice the bracketing with the partitive genitive *Mathēseōs* set between the adjective-noun pair *eam . . . partem*; similarly, the prepositional phrase *in hōc opere* is set between the noun-participle pair *Disquīsītiōnēs . . . contentae*

quae: has the antecedent *partem*

frāctīs plērumque, surdīs semper exclūsīs: *and with fractional (numbers) mostly and irrational (numbers) always excluded*; ablative absolute containing the nouns *frāctīs* and *surdīs*, which can either be treated as substantives or as modifying an implicit *numerīs*

[. . .] Pertinent ad Arithmēticam Sublīmiōrem ea, quae Euclīdēs in Elementīs L. VII sqq. ēlegantiā et rigōre apud veterēs cōnsuētīs trādidit: attamen ad prīma initia huius scientiae līmitantur.

---

arithmētica, -ae f.: arithmetic (from the Greek ἀριθμός for "number")
sublīmis, -e: high
Euclīdēs, -is m.: Euclid (Latinization of Εὐκλείδης; see below)
elementum, -ī n.: element
L. = liber
sqq.: and the following (abbreviation for *et sequentia*)
ēlegantia, -ae f.: elegance
rigor, -ōris m.: (mathematical) rigor
apud (+ acc.): among
veterēs, -um m.: the ancients (from *vetus, veteris*, "old")
cōnsuēscō, -ere, -suēvī, -suētus: be accustomed
trādō, -ere, -didī, -ditus: give up, deliver (both in the sense of "surrendering" or, here, of "passing down" through writing)
attamen: but nevertheless (from *at* + *tamen*)
initium, -iī n.: beginning
scientia, -ae, f.: science (not limited to natural philosophy)
līmitō (1): limit

Arithmēticam Sublīmiōrem: literally "Higher Arithmetic," the traditional name for number theory

ea, quae: *the things which* or *that which*; *ea* is the subject of *pertinent*

Euclīdēs in Elementīs L. VII sqq.: although Euclid's *Elements* is perhaps best known for proving geometrical results axiomatically from 5 postulates, Books VII to X deviate from the work's main focus and instead cover number theory; specifically, VII deals with the fundamentals of divisors and multiples, VIII with geometric sequences, IX with advanced results like Euclid's theorem (that there are infinitely many primes), and X with irrational proportions

ēlegantiā et rigōre apud veterēs cōnsuētīs: *through the elegance and rigor customary among the ancients*; *cōnsuētīs* is plural because it modifies the two ablatives of means *ēlegantiā* and *rigōre* simultaneously; notice the *apud veterēs* bracketed between the noun-participle pair

Diophantī opus celebre, quod tōtum problēmatīs indēterminātīs dicātum est, multās quaestiōnēs continet quae propter difficultātem suam artificiōrumque subtīlitātem dē auctōris ingeniō et acūmine exīstimātiōnem haud mediocrem suscitant, ⇒

Diophantus, -ī m.: Diophantus (Latinization of Διόφαντος; see below)
celeber, -bris, -bre: celebrated
tōtus, -a, -um: (used as a quasi-adverb) wholly; in his/her/its entirety
problēma, -atis n.: problem (from the Greek πρόβλημα, something "put forward")
indēterminātus, -a, -um: indeterminate
dicō (1): dedicate, devote
quaestiō, -ōnis f.: inquiry, question
difficultās, -ātis f.: difficulty
artificium, -iī n.: art, skill
subtīlitās, -ātis f.: subtlety
auctor, -ōris m.: author, writer (the word itself suggests an authority on the subject matter)
ingenium, -iī n.: genius (describing either the quality or a person)
acūmen, acūminis f.: (lit.) point; acuteness, acumen
exīstimātiō, -ōnis f.: judgment, opinion; reputation
haud (adv.): not at all
mediocris, -e: mediocre, unremarkable; (with a negation like *haud*, as an example of litotes) extraordinary
suscitō (1): lift up, elevate, raise

Diophantī opus celebre: Gauss is referencing the *Arithmētica* by the 3rd-century CE Greek mathematician Diophantus, now known as the "father of algebra"; his work focuses on the eponymous Diophantine equations — see below

problēmatīs indēterminātīs: *indeterminate problems*; *problēmatīs* is a variant of the more common dative form *problēmatibus*; an "indeterminate problem" is a system that has fewer equations than unknowns (so there do not exist finitely many solutions, unless additional conditions — like only allowing integer solutions — are added)

quae: the subject of *suscitant*, with *multās quaestiōnēs* as the antecedent

(continued) praesertim sī subsidiōrum quibus illī ūtī licuit tenuitātem cōnsīderēs. At quum haec problēmata dexteritātem quandam potius scītamque tractātiōnem quam prīncipia profundiōra postulent, praetereāque nimis speciālia sint rārōque ad conclūsiōnēs generāliōrēs dēdūcant, ⇒

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praesertim (adv.): especially
subsidium, -iī n.: support, assistance, resource
ūtor, ūtī, ūsus sum (+ abl.): use
licet, -ēre, -uit, – (+ dat. and inf.): (impers.) it is allowed, it is possible
tenuitās, -ātis f.: slenderness; (here) scarcity
cōnsīderō (1): look closely at; reflect upon, consider
quum = cum
dexteritās, -ātis f.: dexterity, cleverness
potius (+ *quam*): rather (than)
tractātiō, -ōnis f.: handling, manipulation
prīncipium, -iī n.: principle
profundus, -a, -um: deep, profound
postulō (1): ask, demand, require
praetereā: in addition, moreover (from *praeter* + *eā (rē)*)
nimis (adv.): too
speciālis, -e: special, specific
rārō (adv.): rarely (adverbial form of *rārus, -a, -um*)
conclūsiō, -ōnis f.: conclusion
generālis, -e: general
dēdūcō, -ere, -dūxī, -ductus: (here) lead

---

praesertim sī . . . cōnsīderēs: *especially if you should consider that . . .* ; the future-less-vivid protasis of a conditional with a present subjunctive verb

quibus illī ūtī licuit: *which it was possible for him to use* or, more elegantly, *which he could use*; *licuit* takes the dative *illī* (referring to the *auctor*, Diophantus) and *ūtī* takes the ablative *quibus*, the antecedent of which is *subsidiōrum*

quum . . . postulent: *postulent*, as well as *sint* and *dēdūcant* later, are subjunctive verbs in a causal *cum* (or, here, *quum*) clause

(continued) hic liber ideō magis epocham in historiā Mathēseōs cōnstituere vidētur quod prīma artis charactēristicae et Algebrae vestīgia sistit, quam quod Arithmēticam Sublīmiōrem inventīs novīs auxerit.

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ideō (adv.): for that reason, therefore (from *id* + *eō*, lit. "this from that")

magis (+ *quam*): rather (than)

epocha, -ae f.: age, epoch (from the Greek ἐποχή, a "fixed point")

historia, -ae f.: history (from the Greek ἱστορία, an "inquiry")

cōnstituō, -ere, -ī, -ūtus: set up, establish

ars, artis f.: art, skill

charactēristicus, -a, -um: characteristic

algebra, -ae f.: algebra (from the Arabic الْجَبْر *al-jabr* for "the setting up," a term used by the mathematician al-Khwarizmi to refer to a method of solving equations)

vestīgium, -iī n.: footprint; mark, trace (cf. vestige)

sistō, -ere, stitī, status: place, set, plant

inventum, -ī n.: invention, discovery (from *inveniō, -īre, -vēnī, -ventus*, "to find")

augeō, -ēre, auxī, auctus: increase, strengthen, advance (cf. auction)

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quod: *because*, and not a relative pronoun

artis charactēristicae: by "characteristic art," Gauss is referring to Diophantus's unprecedented algebraic notation, which he likely appreciated given his own contributions to number-theoretic notation; specifically, in the Diophantine system, a variable is represented by ζ, its square by Δυ (a truncation of δύναμις, "power"), and its cube by Κυ (κύβος, "cube"), and greater exponents by combinations of smaller powers; since this approach never caught on, subsequent mathematicians were forced to employ verbose and geometric descriptions of equations, at least until Viète and Descartes proposed the use of letters for variables and superscripts for exponents, respectively

auxerit: *quod* with a subjunctive expresses an alleged reason

inventīs novīs: ablative of means

Longē plūrima recentiōribus dēbentur, inter quōs paucī quidem sed immortālis glōriae virī P. de Fermat, L. Euler, L. La Grange, A. M. Le Gendre ⇒

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longē (adv.): (here, with superlative) by far

plūrimus, -a, -um: most (superlative of *multus, -a, -um*)

recentiōrēs, -um m.: the moderns (as a substantive of the comparative for *recēns, -entis*, "recent," here referring to the scientists and mathematicians of the Modern era)

immortālis, -e: immortal, eternal

glōria, -ae f.: glory

---

inter quōs paucī quidem sed immortālis glōriae virī: *among whom (these) men, few indeed but of eternal glory*; take *quidem* closely with *paucī*; *immortālis glōriae* is a genitive of description that is treated almost adjectively

P. de Fermat: Pierre de Fermat was a 17th-century French mathematician famous for his conjecture that the equation $a^n + b^n = c^n$ has no positive integer solutions a, b, c for $n > 2$; in a margin of his copy of the aforementioned *Arithmētica* by Diophantus, he wrote that he had found "a truly marvelous proof" (*dēmōnstrātiōnem mīrābilem sānē*) for the conjecture, but claimed that the margin was too small to contain it (*Hanc marginis exiguitās nōn caperet*); the first rigorous proof of Fermat's Last Theorem was developed by Andrew Wiles in 1994, over three centuries later!

L. Euler: in addition to founding graph theory (see the previous excerpt), Leonhard Euler also laid the groundwork for number theory (providing the first proof of Fermat's theorem on sums of two squares, again not proven by Fermat himself!)

L. La Grange: the 18th century mathematician Joseph-Louis Lagrange is perhaps best known as a physicist (having developed Lagrangian mechanics, an alternative to Newtonian mechanics), but he also worked on binary quadratic forms and proved that every positive integer can be expressed as a sum of four squares of nonnegative integers

A. M. Le Gendre: Adrian-Marie Legendre, a contemporary of Gauss, conjectured in 1785 the quadratic reciprocity law proven by Gauss in the *Disquīsītiōnēs*; he is also known for proving Fermat's Last Theorem in the special case of $n = 5$

(continued) (ut paucōs aliōs praeteream) introitum ad penetrālia huius dīvīnae scientiae aperuērunt, quantīsque dīvitiīs abundent patefēcērunt.

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praetereō, -īre, -iī, -itus: pass over, not mention
introitus, -ūs m.: entrance
penetrālis, -e: inner; (in the neuter plural as a substantive) innermost parts
dīvīnus, -a, -um: divine
aperiō, -īre, -uī, -tus: uncover, reveal
quantus, -a, -um: how much, (pl.) how many
dīvitiae, -ārum f.: riches
abundō (1): overflow (with dat.)
patefaciō, -ere, -fēcī, -factus: lay open, expose

---

ut paucōs aliōs praeteream: literally *so that I may pass over a few others* but more idiomatically *not to mention a few others*; a purpose clause

quantīsque dīvitiīs abundent: indirect question with the subjunctive verb *abundent*, whose implicit subject is *penetrālia*

[. . .] Prōpositum huius operis, ad quod ēdendum iam annōs abhinc quīnque pūblicē fidem dederam, id fuit, ut disquīsītiōnēs ex Arithmēticā Sublīmiōrī, quās partim ante id tempus partim posteā īnstituī, dīvulgārem.

---

prōpositum, -ī n.: (here) purpose
ēdō, -ere, -didī, -ditus: put forth, publish (cf. edition)
abhinc (adv.): ago, since (with an acc. of amount of time; from *ab* + *hinc*)
pūblicē (adv.): openly, publicly
fidēs, -eī f.: trust; promise, assurance
partim . . . partim: partly . . . partly
ante (prep. + acc.): before
posteā: thereafter (from *post* + *ea*)
īnstituō, -ere, -ī, -ūtus: set, make, undertake
dīvulgō (1): disseminate, publish

---

ad quod ēdendum . . . fidem dederam: *which I had sworn . . . to publish*; *ad* plus an accusative gerund denotes purpose, and *quod* is the object of the gerund *ēdendum*; *fidem dare* literally translates as "to give assurance," but more idiomatically means "to swear"

id fuit, ut . . . dīvulgārem: *was this, that I might disseminate* or, shorter, *was to disseminate . . .* ; in later Latin, *id* (like *hoc*) is often used to refer to a subsequent clause (here, a noun clause introduced by *ut* and in apposition to *id*)

Nē quis vērō mīrētur scientiam hīc ā prīmīs propemodum initiīs repetītam, multāsque disquīsītiōnēs hīc dēnuō resūmtās esse, quibus aliī operam suam iam nāvārunt, ⇒

---

vērō (adv.): in truth; but, and (postpositive)

mīror, -ārī, -ātus sum: be amazed, be surprised

propemodum (adv.): nearly, almost (from *prope* + *modum*, lit. "close to the limit")

repetō, -ere, -īvī, -ītus: strike again, revisit, undertake again

dēnuō (adv.): anew, again

resūmō, -ere, -psī, -ptus (-tus): take up again

nāvō (1): do enthusiastically; (as an idiom with *operam suam* and dat.) pursue vigorously

---

Nē . . . mīrētur: negative purpose clause, often introduced by "lest"

quis: after *nē*, *quis* is equivalent to *aliquis*

scientiam . . . repetītam, multāsque disquīsītiōnēs . . . resūmtās esse: assume an *esse* to complete the infinitive *repetītam esse*, which, along with *resūmtās esse*, is an infinitive in an object clause (constructed identically to an indirect statement) set up by *mīrētur*

nāvārunt: syncopation of *nāvāvērunt*

(continued) monendum esse dūxī mē, quum prīmum initiō a. 1795 huic disquīsītiōnum generī animum applicāvī, omnium quae quidem ā recentiōribus in hāc arēnā ēlabōrāta fuerint ignārum, omniumque subsidiōrum per quae dē hīs quidpiam comperīre potuissem expertem fuisse.

---

moneō, -ēre, -uī, -itus: warn
dūcō, -ere, dūxī, ductus: (here) think
prīmum (adv.): first
a. = annī
applicō (1): (here, with *animum* and dat.) devote one's mind to
arēna = harēna, -ae f.: sand → sand-covered arena (of an amphithetre) → battlefield (in an area of study)
ēlabōrō (1): labor on, take pains with, work out
ignārus, -a, -um (+ gen.): ignorant (of)
quispiam, quaepiam, quidpiam: anyone, anything
comperiō, -īre, -ī, -tus: learn, discover
expers, expertis (+ gen.): devoid of, lacking knowledge of

---

monendum esse dūxī: *I thought that it had to be warned that*; the passive periphrastic *monendum esse* introduces an indirect statement with the subject *mē*, the infinitive *fuisse*, and the predicates *ignārum* and *expertem*

initiō: an ablative of time when

omnium quae . . . ēlabōrāta fuerint ignārum: *ignorant of all that had been worked out . . .* ; *ēlabōrāta fuerint* is a common alternative form of *ēlabōrāta erunt*, which is not a future perfect passive indicative but rather a perfect passive subjunctive (relative clauses in indirect statements require the subjunctive)

per quae dē hīs quidpiam comperīre potuissem: *through which I would have been able to learn anything concerning these things*; the antecedent of *quae* is *subsidiōrum*

Scīlicet in aliō forte labōre tunc occupātus, cāsū incidī in eximiam quandam vēritātem arithmēticam (fuit autem nī fallor theōrēma art. 108), ⇒

---

scīlicet (adv.): indeed, in fact
forte (adv.): by chance
occupātus, -a, -um: occupied, busy
cāsū (adv.): by chance
incidō, -ere, -ī, –: fall into, encounter
eximius, -a, -um: select, extraordinary
vēritās, -ātis f.: truth
nī = nisi
fallō, -ere, fefellī, falsus: (here, in the pass.) be mistaken
theōrēma, -atis n.: theorem (from the Greek θεώρημα, a "sight" or "speculation")
art. = articulus, -ī m.: part, section (referring to the numbered paragraphs in Gauss's book)

---

fuit autem nī fallor theōrēma art. 108: *nī fallor* is a brief conditional meaning "if I am not mistaken"; Gauss is referring to the following theorem, a corollary to Euler's criterion: -1 is a quadratic residue of all prime numbers of the form $4n + 1$, but a nonresidue of all prime numbers of the form $4n + 3$; in other words, there exists some x such that $x^2 \equiv -1 \pmod{p}$ for a prime $p > 2$ if and only if $p \equiv 1 \pmod{4}$ (see below for Gauss's own explanation of modular arithmetic); this lemma is one of the earliest steps towards the proof of Gauss's famous quadratic reciprocity law

(continued) quam quum et per sē pulcherrimam aestimārem et cum maiōribus connexam esse suspicārer, summā quā potuī contentiōne in id incubuī ut prīncipia quibus innīterētur perspicerem, dēmōnstrātiōnemque rigōrōsam nancīscerer.

---

per sē: for itself, in itself
aestimō (1): value, appraise; judge
connectō = cōnectō, -ere, -nexuī, -nexus: connect, relate
suspicor, -ārī, -ātus sum: suspect, believe
contentiō, -ōnis f.: straining, effort
incubō, -āre, -uī, -itus: (here) apply oneself vigorously
innītor, -ī, -nīxus sum (+ dat.): rest on
perspiciō, -ere, -spexī, -spectus: look closely at, inspect; ascertain
dēmōnstrātiō, -ōnis f.: demonstration, proof
rigōrōsus, -a, -um: (mathematically) rigorous
nancīscor, -ī, nactus sum: obtain

---

quam quum . . . aestimārem et . . . suspicārer: the antecedent of *quam* is *vēritātem*; a connective to a previous clause or sentence (in this case, the relative pronoun *quam*) can sometimes precede a conjunction like *quum* even if it is a part of the subsequent clause; *quam* functions as the object of *aestimārem* and the subject of *connexam esse*; the verbs are subjunctive because they reside in a circumstantial *cum* (or *quum*) clause

summā quā potuī contentiōne: *with the greatest effort with which I was able to* or *that I was capable of*; the noun-adjective pair bracketing the relative clause is an ablative of manner, as is the relative pronoun *quā* itself; *potuī* has an implicit complementary infinitive *incubāre*

in id . . . ut: again, as a demonstrative, *id* refers to the subsequent noun clause that is in apposition to it; as a compound verb, *incubāre* would be expected to take a dative, but its prefix can often be repeated in the form of a preposition

Quod postquam tandem ex vōtō successisset, illecebrīs hārum quaestiōnum ita fuī implicātus ut eās dēserere nōn potuerim; quō pactō, dum alia semper ad alia viam sternēbant, ea quae in quātuor prīmīs Sectiōnibus huius operis trāduntur ad maximam partem absolūta erant, antequam dē aliōrum geōmetrārum labōribus similibus quidquam vīdissem.

---

postquam: after
tandem: at last, finally
ē/ex (prep. + abl.): (here) in accordance with
vōtum, -ī n.: vow, promise
succēdō, -ere, -cessī, -cessus: (here, in the 3rd person) be successful
illecebra, -ae f.: enticement, charm
implicō (1): (here with dat.) entangle, involve, engage with (cf. be implicated)
dēserō, -ere, -uī, -tus: leave, abandon
quō pactō: in which/this way (equivalent to *quōmodo*)
dum: (as a later Latin substitution for *cum* that may use the indic.) since
sternō, -ere, strāvī, strātus: (here, with *viam*) pave
quātuor = quattuor
sectiō, -ōnis f.: (here, of a book) section
absolvō, -ere, -ī, -solūtus: release; (here) complete
antequam: before
geōmetrēs, -ae m.: geometer, scholar of geometry (from the Greek γεωμέτρης, "earth-measurer")
similis, -e: similar
quisquam, quidquam: anyone, anything

Quod: the antecedent of this relative pronoun is *id* and the associated noun clause above; *quod* is the subject of *successisset* and should be translated as a demonstrative ("this"); again, as *quod* connects back to the previous sentence, it can precede the clause set up by the conjunction *postquam* even if it belongs to it

fuī implicātus: equivalent to *implicātus sum*

ita . . . ut: *so . . . that*; *ita* anticipates a result clause

alia . . . ad alia: *some things . . . to other things*

ea: subject of *absolūta erant*

ad maximam partem: *for the most part*

dē aliōrum geōmetrārum labōribus similibus quidquam: *anything concerning similar efforts of other geometers*

vīdissem: the subjunctive expresses the hypotheticality of the statement (i.e. even once Gauss had access to the works of contemporary number theorists, he could have simply stumbled across them by chance)

Dein cōpiā mihi factā hōrum summōrum ingeniōrum scrīpta ēvolvendī, maiōrem quidem partem meditātiōnum meārum rēbus dūdum trānsāctīs impēnsam esse agnōvī: sed eō alacrior, illōrum vestīgiīs īnsistēns, Arithmēticam ulterius excolere studuī.

---

cōpia, -ae f.: (here) opportunity
scrīptum, -ī n.: writing, work (from *scrībō, -ere, scrīpsī, scrīptus*)
ēvolvō, -ere, -ī, -volūtus: roll; read (i.e. roll open a scroll), peruse
meditātiō, -ōnis f.: contemplation
dūdum (adv.): a while ago, some time ago
trānsigō, -ere, -ēgī, -āctus: (here) settle
impendō, -ere, -ī, -pēnsus: weigh out; expend, devote
agnōscō, -ere, -nōvī, -nitus: recognize
eō (adv.): therefore, for that reason
alacer, -cris, -cre: eager
īnsistō, -ere, -stitī, – (+ dat.): stand upon, follow
ulterius (adv.): further
excolō, -ere, -uī, -cultus: work out carefully, improve, develop
studeō, -ēre, -uī, – (+ inf.): be eager, take pains, strive

---

cōpiā mihi factā, . . . scrīpta ēvolvendī: *cōpiā . . . factā* is an ablative absolute, and *mihi* is simply a dative of reference; *facere cōpiam* is an idiom meaning "to give an opportunity"; *cōpiā* takes the genitive gerund *ēvolvendī*, which has the object *scrīpta*

rēbus dūdum trānsāctīs: *to matters settled some time ago*

Sectiō Prīma: Dē Numerōrum Congruentiā In Genere

1. Sī numerus a numerōrum b, c differentiam mētītur, b et c *secundum* a *congruī* dīcuntur, sīn minus, *incongruī*: ipsum a *modulum* appellāmus. Uterque numerōrum b, c priōrī in cāsū alterīus *residuum*, in posteriōrī vērō *nōnresiduum* vocātur.

---

congruentia, -ae f.: (in math) congruence
differentia, -ae f.: difference
mētior, -īrī, mēnsus sum: measure; measure into portions, divide (without any remainder)
secundum (prep. + acc.): with respect to
congruus, -a, -um: congruent
sīn: but if
minus: (here) not
incongruus, -a, -um: incongruent
modulus, -ī m.: measuring unit; modulus
appellō (1): name
uterque, utraque, utrumque (+ gen.): both
prior, prius first, former
cāsus, -ūs m.: case
residuum, -ī n.: residue
posterior, -ius: latter (of two)
nōnresiduum, -ī n.: nonresidue

In Genere: an idiom for "in general"

Sī numerus . . . vocātur: in other words, if $b-c$ is a multiple of a (more generally, $b = ka + c$ for some integer k), then b is congruent to c modulo a, and b and c are residues of each other; Gauss will soon introduce the notation $b \equiv c \pmod{a}$ for this congruence relation, the basis of modular arithmetic

b et c *secundum* a *congruī* dīcuntur: *b and c are called congruent with respect to a*; the variable a is in the accusative

ipsum a: in math, *ipse* is often used to indicate the grammatical case of a variable and can remain untranslated

Uterque numerōrum b, c . . . vocātur: here Gauss highlights one of the three essential properties of an equivalence relation, namely symmetry: b is a residue of c modulo a if and only if c is a residue of b modulo a (in other words, $c \equiv b \pmod{a}$ if and only if $b \equiv c \pmod{a}$); the other two properties are reflexivity and transitivity, explained on the next page

priōrī in cāsū: reorder as *in priōrī cāsū* for clarity

in posteriōrī: insert *cāsū*

Hae nōtiōnēs dē omnibus numerīs integrīs tam positīvīs quam negātīvīs* valent, neque vērō ad frāctōs sunt extendendae. E.g. -9 et $+16$ secundum modulum 5 sunt congruī; -7 ipsīus $+15$ secundum modulum 11 residuum, secundum modulum 3 vērō nōnresiduum. Cēterum quoniam cifram numerus quisque mētītur, omnis numerus tamquam sibi ipsī congruus secundum modulum quemcunque est spectandus.

*Modulus manifestō semper *absolūtē* i.e. sine omnī signō est sūmendus.

---

nōtiō, -ōnis f.: (here) notion
tam . . . quam: as . . . as; (here) both . . . and
positīvus, -a, -um: positive
negātīvus, -a, -um: negative
valeō, -ēre, -uī, -ītus: (here) hold true
extendō, -ere, -ī, -tentus (-tēnsus): extend
cēterum (adv.): (here) in addition, moreover
quoniam: since
cifra, -ae f.: number (cf. French "chiffre"); zero (from the Arabic word صِفْر *ṣifr*)
tamquam: as much as; like, as
quīcumque, quaecumque, quodcumque (-cunque): whoever, whatever; every, any
spectō (1): look at, observe; regard, consider
manifestō (adv.): clearly
absolūtē (adv.): absolutely
signum, -ī n.: sign
sūmō, -ere, -psī, -ptus: take

---

dē . . . valent *hold true with respect to* or *for*

ad frāctōs sunt extendendae: passive periphrastic; *frāctōs* modifies an implicit *numerōs*; technically, in some applications, fractions are permissible in modular arithmetic, so, for instance, $7/2 \equiv 3/2 \pmod 2$ or $3/4 \equiv 1/4 \pmod{1/2}$

Cēterum quoniam cifram . . . est spectandus: Gauss here demonstrates the reflexivity of the congruence relation by explaining why $a \equiv a \pmod m$ for any modulo m

sibi ipsī: *to its very self*; *ipsī* is used to emphasize *sibi*

est spectandus: another passive periphrastic

Modulus . . . est sūmendus: a passive periphrastic; here, Gauss clarifies that the sign of the modulus is irrelevant, as $-a$ has the same multiples as a

2. Omnia numerī datī a residua secundum modulum m sub fōrmulā $a + km$ comprehenduntur, dēsignante k numerum integrum indēterminātum. Prōpositiōnum quās post trādēmus faciliōrēs nūllō negōtiō hinc dēmōnstrārī possunt: sed istārum quidem vēritātem aequē facile quīvīs intuendō poterit perspicere.

---

datus, -a, um: given
sub (prep. + abl.): (here) by means of, through
fōrmula, -ae f.: formula
comprehendō, -ere, -ī, -hēnsus: (here) express
dēsignō (1): designate, represent
prōpositiō, -ōnis f.: proposition
negōtium, -iī n.: (here) difficulty, trouble
hinc: from this place; (here) hence
dēmōnstrō (1): demonstrate, prove
aequē (adv.): equally, just as
facile (adv.): easily (adverbial form of *facilis, -e*)
quīvīs, quaevīs, quidvīs: whoever, whatever; anyone, anything
intueor, -ērī, -itus sum: look at

Omnia numerī datī a **residua:** *all residues of a given number a*; note the bracketing

dēsignante k numerum integrum indēterminātum: *with k representing an indeterminate* (in modern mathematical language, "some") *integer*; ablative absolute where the variable k is the noun, *dēsignante* is the participle, and *numerum* is the object

Prōpositiōnum: a partitive genitive governed by *faciliōrēs*

istārum: refers back to *prōpositiōnum*

intuendō: *by looking*; gerund

Numerōrum congruentiam hōc signō, $\equiv$, in posterum dēnotābimus, modulum ubi opus erit in clausulīs adiungentēs, $-16 \equiv 9 \pmod 5$, $-7 \equiv 15 \pmod{11}$*.

*Hoc signum propter magnam analogiam quae inter aequālitātem atque congruentiam invenītur adoptāvimus. Ob eandem caussam ill. Le Gendre in comment. īnfrā saepius laudandā ipsum aequālitātis signum prō congruentiā retinuit, quod nōs nē ambiguitās oriātur imitārī dubitāvimus.

---

in posterum: (idiom) in the future
dēnotō (1): denote
opus est: it is needed
clausula, -ae f.: parenthesis (i.e. an "enclosing" character)
adiungō, -ere, -iūnxī, -iūnctus: join, add
analogia, -ae f.: analogy
aequālitās, -ātis f.: equality
inveniō, -īre, -vēnī, -ventus: find
adoptō (1): choose, adopt
ob (prep. + acc.): on account of, because of
caussa = causa
ill. = illūstris, -e: illustrious
comment. = commentātiō, -ōnis f.: treatise
īnfrā (adv.): below
saepius (adv.): more often, very often (comparative of *saepe*)
laudō (1): (here) name, cite
retineō, -ēre, -uī, -tentus: retain
ambiguitās, -ātis f.: ambiguity
orior, -īrī, -tus sum: arise
imitor, -ārī, -ātus sum: imitate
dubitō (1): hesitate

---

hōc signō: ablative of means; Gauss introduced the symbol $\equiv$ into number theory

modulum ubi opus erit in clausulīs adiungentēs: *adding the modulus in parentheses when it is needed* or, shorter, *when needed*; the participle *adiungentēs* is modifying the subject of the sentence, "we"

in comment. īnfrā saepius laudandā: *in a treatise to be cited very often below* elsewhere in Gauss's work, namely Legendre's 1798 *Essai sur la Théorie des Nombres* ("Essays on the Theory of Numbers")

ipsum aequālitātis signum: the pronoun *ipsum* serves to bracket the appositional genitive *aequālitātis*

quod: the antecedent for this neuter relative pronoun is the entire previous clause

nē ambiguitās oriātur: a negative purpose clause; for similar reflections on notation, see the second excerpt for Peano in this reader

3. Theōrēma. Prōpositīs m numerīs integrīs successīvīs $a, a+1, a+2, \ldots, a+m-1$ aliōque A illōrum aliquis huic secundum modulum m congruus erit, et quidem ūnicus tantum.

Sī enim $\frac{A-a}{m}$ integer, erit $a \equiv A$, sīn frāctus, sit integer proximē maior (aut quandō est negātīvus, proximē *minor*, sī ad signum nōn respiciātur) $= k$, cadetque $A+km$ inter a et $a+m$, quārē erit numerus quaesītus.

---

prōpōnō, -ere, -posuī, -positus: set
successīvus, -a, -um: successive, consecutive
ūnicus, -a, -um: one
tantum: only
proximē (adv.): next
quandō: when
respiciō, -ere, -spexī, -spectus (+ *ad*): look at; have regard for
cadō, -ere, cecidī, cāsus: fall
quārē: for which reason, therefore

Prōpositīs *m* numerīs integrīs successīvīs . . . aliōque *A*: ablative absolute with the noun phrases m *numerīs integrīs successīvīs* and *aliō* A

illōrum aliquis huic: *illōrum* (plural), "the former," refers to the m consecutive integers, while *huic* (singular), "the latter," refers to the one integer A; Gauss emphasizes that there exists some integer, and "indeed only one" (*quidem ūnicus tantum*), between a and $a+m-1$ that is congruent to A; this theorem implies the final property of the congruence relation, transitivity, concerning which he later writes, "If many numbers are congruent to the same number with respect to the same modulus, they are congruent between themselves" as well (*Sī plūrēs numerī eīdem numerō secundum eundem modulum sunt congruī, inter sē erunt congruī*); i.e., if $a \equiv b \pmod{m}$ and $c \equiv b \pmod{m}$, then $a \equiv c \pmod{m}$

Sī enim $\frac{A-a}{m}$ integer: assume the verb *est*; do the same for the next conditional, *sīn frāctus*

sit integer proximē maior . . . = *k*: *let the next greater integer* $= k$ or *be k*; *sit* is very commonly used when defining variables

sī ad signum nōn respiciātur: the future-less-vivid protasis of a conditional with a present subjunctive verb that is usually translated with "should"

***A* + *km*:** subject of *cadet*

Et manifestum est omnēs quotientēs $\frac{a-A}{m}$, $\frac{a+1-A}{m}$, $\frac{a+2-A}{m}$, etc. inter $k-1$ et $k+1$ sitōs esse; quārē plūrēs quam ūnus integrī esse nequeunt.

---

manifestus, -a, -um: evident, clear (cf. manifest)

quotiēns, -entis m.: quotient

situs, -a, -um: (here, with *esse*) lie, be situated

plūs, plūris: more (comparative of *multus, -a, -um*; in the singular, usually only neuter)

nequeō, -īre, -īvī, -itus: cannot

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omnēs quotientēs . . . sitōs esse: indirect statement; the reasoning in the $a \not\equiv A$ case is the following: since k is defined as the first integer greater than (i.e. the ceiling of) the non-integer fraction $\frac{a-A}{m}$, k must lie strictly between $\frac{a-A}{m}$ and $\frac{a-A}{m}+1$; therefore, $A+km$ must lie strictly between $A+m\cdot\frac{a-A}{m} = A+a-A = a$ and $A + m \cdot (\frac{a-A}{m} + 1) = A + a - A + m = a + m$, and since $A + km$ is an integer, $A + km$ must be equal to one (and exactly one) of the integers between $a+1$ and $a+m-1$, inclusive, implying that A is congruent to that same integer

Chapter 10 — Peano: "Dē Latīnō Sine Flexiōne" & "Prīncipiō Dē Permanentia"

At the turn of the 20th century, in a fierce metaphysical debate among the mathematical community on the limitations of scientific knowledge and logic itself, Emil du Bois-Reymond encapsulated his stance with the word *ignōrābimus*, "we will not know." The use of a Latin term may come as a surprise, considering that German and French had already largely supplanted Latin as the common languages of academics for almost a century. To counter du Bois-Reymond's perspective, the mathematician David Hilbert advocated for a more optimistic outlook, albeit without the Latin flourish, stating in an influential 1900 lecture that "[i]nstead of the foolish Ignorabimus, our watchword should be, on the contrary, 'We must know. We will know'" (McCarty, 2005).[1] Specifically, Hilbert anticipated in the lecture that mathematicians would eventually develop a finite set of basic assumptions or axioms that would be able to prove every true logical statement (a property known as completeness), never lead to contradictory results (consistency), and even determine the existence of a solution to any math problem (decidability).

An early contributor in the movement to axiomatize all of mathematics was the Italian mathematician Giuseppe Peano (1858 CE – 1932 CE). Before him, little advancement had been made since Euclid's *Elements*, which presented the five postulates defining flat, two-dimensional geometry and the rules for straightedge and compass constructions; indeed, for millennia, the *Elements* was considered the paragon of an axiomatic system and used as the basis for

[1]The original German version of this phrase, "Wir müssen wissen / Wir werden wissen," is inscribed on Hilbert's gravestone.

proof within even algebra and number theory. However, Peano produced examples of geometrical objects that tested the limits of Euclidean geometry (e.g. the Peano curve; see Figure 1 below) and raised doubts on its reliability as a foundation for the rest of mathematics, doubts that remain relevant even after revisions of the Euclidean axioms by Hilbert and others. Peano also set his sights on fields other than geometry that were in need of a sound, axiomatic formulation, in particular arithmetic. Instead of defining the binary operation of addition, Peano presented in 1889 the more basic notion of a successor operation, which transforms a natural number (e.g. 3) into its successor (4). The five Peano axioms for arithmetic define a starting number 1 that is not the successor of any other number and that, through repeated application of the successor operation, generates all natural numbers. In contrast to Richard Dedekind, who had attempted to derive the natural numbers from set theory a year earlier, Peano recognized that the relations between objects in axiomatic mathematics are a more suitable focus of study than the nature of the objects themselves, which is undefinable (Segre, 1994).

Peano wrote the *Arithmēticēs prīncipia, novā methodō exposita*, the work in which he first introduced his eponymous axioms, in Latin (although the Peano axioms themselves were of course expressed in exclusively set theoretic notation). This decision reflects Peano's interests beyond mathematics and especially his study of Leibniz's work. As described in more detail in the chapter devoted to the 17th-century philosopher, Leibniz had envisioned a *charactēristica ūniversālis* or "universal language" that not only would be comprehensible to anyone, but could also be used to verify whether a given logical statement in the language is true through computation alone. In the spirit of such an ideal *charactēristica ūniversālis*, Peano paid sig-

nificant attention to developing and adopting proper mathematical notation (Luciano, 2012). For example, he invented the symbol ϵ, as an abbreviation of *est* to represent inclusion in a set, which survives to this day as the symbol $\in$. Similarly, the idea of being able to verify the validity of statements has now been partially realized in the form of mathematical proof assistant software, which in fact rely on Peano's axioms. More importantly for the excerpt below, however, Peano also understood the need for an artificial language for international communication between mathematicians (Aray, 2021). To that end, he designed the *Latīnō sine flexiōne* or "Latin without inflections." In his 1903 article, "Dē Latīnō Sine Flexiōne: Lingua Auxiliare Internationale," excerpted here, he takes a creative approach to teaching his new language, initially writing in Latin, before transitioning to the "Latin without inflections" through the gradual removal of grammatical features he declares superfluous for scientific texts, such as gender, number, or even tense. After 1908, Peano tried popularizing the *Latīnō sine flexiōne* through his position as the director of the "Akademi internasional de lingu universal," and became an active participant in the International Auxiliary Language community generally, which in the present day is most famous for producing Esperanto. As a disclaimer, these "international" languages were created partially in response to growing tension between powerful European nations and often relied on Germanic or Romance vocabulary to foster a European identity (Aray, 2021), ignoring the needs of members of other language families, who would obviously have a disproportionally difficult experience learning these languages. As a matter of fact, Peano began to undertake the mission of constructing the "Latin without inflexions" upon receiving an article written in Japanese (a product of Japan's rapid globalization in the late 19th century)

(Ostler, 2007), although he did appreciate the lack of inflectional grammar in Chinese (Aray, 2021).

As a proof of concept, Peano authored a short companion article titled "Prīncipiō Dē Permanentia", which, as its subtitle "Exercitiō dē Latīnō rēctō" indicates, was intended to be a relatively straightforward philosophical paper (with a handful of citations written in their original language juxtaposed with their translation in the "Latin without inflexions"). After summarizing the rules explicated in "Dē Latīnō Sine Flexiōne," Peano references and dismisses a definition for the "principle of permanence" proposed by the mathematician Hermann Schubert, that the notation for one number system that is reused in another should signify that the two systems share the same rules or axioms. Since Peano believes that such a notion is far too limiting (as it would imply that one cannot use the same inequality symbol $<$ for positive and negative numbers, for instance), in the second excerpt below he advances his own version of the "principle" — or, rather, recommendation — "of permanence," adapted from the "economy of thought" by the Austrian philosopher and physicist Ernst Mach. By briefly revisiting the history of algebra since the time of the Ancient Greeks, he advises that a new scientific or mathematical discovery should be expressed in the way that is most accessible to the brain of a learner — Mach had argued that this principle stems from the very biological needs of the human organism (Mach, 1903). For instance, rather than inventing completely new notation for vector calculus for the sake of "logical consistency," mathematicians should preserve the old algebraic notation to avoid overwhelming students. Again, Peano emphasizes that the principle of permanence is "a principle, not of logic, but of practice": Although the notation between algebra and vector calculus may be identical,

if his suggestions are followed, there would still not exist a one-to-one logical correspondence between the operations +, −, etc. in each field (Toader, 2021). While the *Latīnō sine flexiōne* may have become a "dead" language like Latin itself following Peano's own death, the logic and clarity it represents remain central to modern mathematical writing, in whatever language it may be communicated.

Source Texts: Our Latin texts come from Peano (1903a) and Peano (1903b).

Note: The vocabulary entries below are in standard Latin. Indeed, Peano created the *Latīnō sine flexiōne* in such a way that anyone can read or write in the language by only consulting the principal parts provided in Latin dictionaries.

Peano's Space-Filling Curves

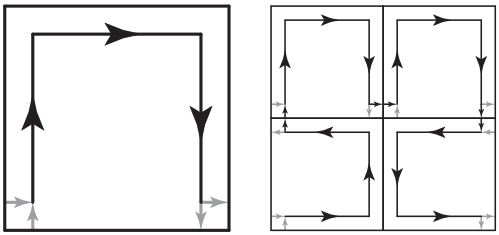
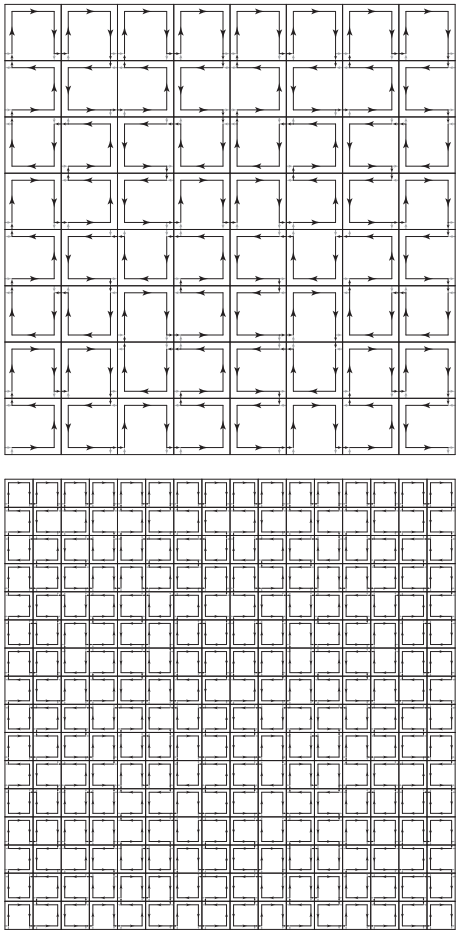


Figure 1: Peano's attention to rigor and fundamental assumptions was not merely useful for his creation of some axiomatic systems, but also for his dismantling of others through counterexamples defying intuition. The Peano curve is one such example meant to specifically question the definition of "dimensionality." In a curve, every point can be identified using one parameter, e.g. the distance traveled along the line, while each point in a plane would appear to require two parameters or coordinates to be located. Peano, however, demonstrated that there exists a curve that traverses each point within the square, thereby complicating the distinction between one-dimensional curves and two-dimensional planes.

The most straightforward construction of a Peano curve is due to David Hilbert: The first step involves simply creating a U curve (shown in the topmost panel) within a square in order to connect the entrance to the tile on the bottom-left corner to the exit on the bottom-right corner. Rotations allow us to connect any two adjacent corners on the square. This tile can now be divided into four smaller squares, which can each contain their own smaller copy of the previously constructed U curve in such a way that the entrance and exit corners of adjacent tiles match. We can now replace each U curve within the smaller squares with this new version, resulting in an even more dense curve filling in the unit square. Two iterations are shown on the right. Repeating this substitution process infinitely, we obtain, in the limit, a one-dimensional curve that, paradoxically, fills a two-dimensional space. ⇒

Figure 1 (continued): This discovery by Peano anticipated the development of the Hausdorff dimension, a generalization of the usual notion of dimension that accounts for the extent to which self-similar structures known as fractals occupy a higher-dimensional space. While the Hausdorff dimension of a straight line is simply 1, the Peano curve has a Hausdorff dimension of 2. There are also various fractals with an intermediary Hausdorff dimension, such as the Koch snowflake with a dimension of $\frac{\ln 4}{\ln 3} \approx 1.26$ (Pickover, 2009).



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Dē Latīnō Sine Flexiōne: Lingua Auxiliāre Internātiōnāle

Lingua latīna fuit internātiōnālis in omnī scientiā, ab imperiō Rōmānō, usque ad fīnem saeculī XVIII. Hodiē multī reputant illam nimis difficilem esse, iam in scientiā, magis in commerciō. Sed nōn tōta lingua latīna est necessāria; parva pars sufficit ad exprimendam quamlibet ideam.

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latīnus, -a, -um: Latin; (as a substantive) the Latin language
flexiō, -ōnis f.: change; (here, in grammar) inflexion
auxiliāris, -e: auxiliary
internātiōnālis, -e: international
scientia, -ae, f.: science
imperium, -iī n.: empire
Rōmānus, -a, -um: Roman
usque (adv.): all the way
fīnis, -is m.: end
saeculum, -ī n.: century
hodiē (adv.): today, in the present
reputō (1): (here) regard, consider
nimis (adv.): too
magis (adv.): (here) more so, to a greater extent
commercium, -iī n.: commerce
necessārius, -a, -um: necessary
sufficiō, -ere, -fēcī, -fectus: (here) be adequate, be sufficient
exprimō, -ere, -pressī, -pressus: (here) express
quīlibet, quaelibet, quodlibet: whatever; any, every
idea, -ae f.: idea (from the Greek ἰδέα, a "form")

Dē Latīnō Sine Flexiōne: Lingua Auxiliāre Internātiōnāle: the title is written in Peano's "Latin without inflections" (the rules of which he explains in the text itself) and, when converted into Latin following his rules below, reads *Latīnae Sine Flexiōne: Lingua Auxiliāris internātiōnālis*; *latīnō* is not an ablative, but rather a genitive because it follows the preposition *dē*, which, in Peano's language, has a breadth of meaning comparable to the French "de" or Italian "di"

reputant illam nimis difficilem esse: *consider that (language) to be too difficult*; *esse* is a complementary infinitive

ad exprimendam quamlibet ideam: *to express every idea*; a gerund or gerundive phrase with *ad* signifies purpose

§ 1. — *Cāsūs*.
« Nōminum cāsūs semper ēlīminārī possunt substitūtīs in eōrum locum particulīs quibusdam. »
LEIBNIZ. Ēd. Couturat a. 1901, p. 67.

Lingua latīna exprimit nōminum cāsūs cum praepositiōnibus « *dē*, *ad*, *ab*, *ex*, … » et cum postpositiōnibus vel dēsinentiīs. Prīma methodus sufficit; ipsa sōla invenītur in latīnō populāre, ā quō dērīvant linguae neolatīnae, ut italica, franca, hispānica, etc.

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cāsus, -ūs m.: (here) grammatical case
nōmen, nōminis n.: name; noun
ēlīminō (1): drive out, eliminate
substituō, -ere, -ī, -ūtus: substitute
particula, -ae f.: grammatical particle (here in the more general sense of a function word, which may include prepositions)
Ēd. = ēditiō, -ōnis f.: edition
a. = annō
p. = pāgina, -ae f.: page
cum (prep. + abl.): (here) with, by means of
praepositiō, -ōnis f.: preposition
postpositiō, -ōnis f.: postposition
dēsinentia, -ae f.: (here, case) ending
methodus, -ī f.: method (from the feminine Greek μέθοδος)
inveniō, -īre, -vēnī, -ventus: find
populāris, -e: of the people, popular, vernacular
dērīvō (1): (here, in linguistics) derive
neolatīnus, -a, -um: Neolatin (here referring to Romance languages)
ut: (here) for example
italicus, -a, -um: Italian
francus, -a, -um: Frankish; French
hispānicus, -a, -um: Spanish

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substitūtīs in eōrum locum particulīs quibusdam: ablative absolute with PPP *substitūtīs*; *in eōrum locum* is an expression for “in(to) their place”

LEIBNIZ. Ēd. Couturat a. 1901, p. 67.: the mathematician Louis Couturat, a close correspondent with Peano, published *La Logique de Leibniz* in 1901 and provided commentary on Leibniz’s thoughts on philosophy and, in particular, on the *charactēristica ūniversālis*, which he himself also attempted to realize; he contributed to the international auxiliary language of Ido, a variant of the more familiar Esperanto (Luciano, 2012)

latīnō populāre: Peano has in mind what used to be called Vulgar Latin, which was hypothesized in the 19th century as a common ancestor of the Romance languages and treated as cleanly distinct from literary Latin

Sūmimus nōmen īnflexibile sub fōrmā simpliciōre, quae est ablātīvus, vel nōminātīvus, vel alia. Indicāmus genitīvō cum *dē*, datīvō cum *ad*, ablātīvō cum *ab*, *ex*, … Accūsātīvō indicātur cum cōnstrūctiōne, ut in linguīs neolatīnīs, scīlicet cum seriē: nōminātīvō—verbō—accūsātīvō, vel cum seriē: *quī*-accūsātīvō—nōminātīvō—verbō.

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sūmō, -ere, -psī, -ptus: take, receive; (here) adopt

īnflexibilis, -e: inflexible, unchangeable; (here) indeclinable

sub (prep. + abl.): (here) by means of, through

fōrma, -ae f.: form

simplex, simplicis: simple

ablātīvus, -ī m.: ablative (the case pertaining to "something taken away")

nōminātīvus, -ī m.: nominative (the case pertaining to "naming")

indicō (1): indicate

genitīvus, -ī m.: genitive (the case pertaining to "birth", a Latin reinterpretation of the Greek term γενική, which describes the case pertaining to the "classes" that things might belong to)

datīvus, -ī m.: dative (the case pertaining to "giving")

accūsātīvus, -ī m.: accusative (the case pertaining to "accusing"; Latin grammaticians mistranslated the Greek αἰτιατική, meaning the case pertaining to "the effect" of an action, because they thought it derived from αἴτιος, -α, -ον, meaning "responsible for")

cōnstrūctiō, -ōnis f.: (here) grammatical construction (e.g. word order)

scīlicet (adv.): (here) namely

seriēs, -ēī f.: series, order

verbum, -ī n.: (here) verb

Indicāmus genitīvō: Peano begins to transition from standard Latin to his "Latin without inflections," so, from now on, all words that appear to be ablatives (or, likewise, nominatives) can actually represent any case at all

***quī*-accūsātīvō—nōminātīvō—verbō:** Peano's prescribed word order for a question starting with an interrogative pronoun, inspired by the constructions found in many Romance languages and even English

Vocābulāriō latīnō commūne continet nōminātīvō et genitīvō dē nōmen. Rēgula commoda haec est:

« Sūmimus nōmen īnflexibile
« a) aut identicō ad nōminātīvō,
« b) aut nōminātīvō, mūtāta dēsinentia *-us, -um, -ū, -ēs*
« in *-ō, -ō, -ō, -ē*,
« c) aut genitīvō, mūtāta dēsinentia *-ī* in *-ō*, *-is* in *-e*.
« d) ad nōminātīvō *ego*, *tū*, *aliquis*,
« respondē (ablātīvō) *mē*, *tē*, *aliquō*. »

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vocābulārium, -iī n.: vocabulary
commūnis, -e: accessible, common
contineō, -ēre, -uī, -tentus: (here) contain
rēgula, -ae f.: rule (cf. regular)
commodus, -a, -um: suitable, convenient
identicus, -a, -um (+ ad): identical to
mūtō (1): change
respondeō, -ēre, -ī, -spōnsus: (here, with acc. and dat.) correspond to, equate to

---

Vocābulāriō latīnō commūne . . . dē nōmen: in other words, the grammatical entry for a Latin noun presents the nominative and genitive (as is the case throughout this reader!)

mūtāta dēsinentia *-us, -um, -ū, -ēs* in *-ō, -ō, -ō, -ē*: ablative absolute with PPP *mūtāta* and subject *dēsinentia*, which do not look like ablatives (note the lack of a macron) because, according to rule a), the inflexions of nouns in the 1st declension are all reduced to the nominative singular; the singular and plural forms of *dēsinentia* and any other word are indistinguishable and, if necessary, are distinguished with a quantitative adjective

***-ū*:** Peano makes sure to account for this rare nominative singular ending that is exclusive to neuter 4th-declension nouns, even if it only occurs in these five words: *cornū*, “horn”; *genū*, “knee”; *gelū*, “frost”; *pecū*, “cattle”; and *verū*, a ”spit” or “javelin”

ad nōminātīvō *ego*, *tū*, *aliquis*, respondē (ablātīvō) *mē*, *tē*, *aliquō*: *equate (the ablatives)* mē, tē, aliquō *to the nominatives* ego, tū, aliquis; recall that *ad* signifies a dative in Peano’s language; Peano is involving the reader directly in the gradual conversion of Latin into the *Latīnō sine flexiōne* by using the singular imperative verb *respondē*

Rēgula a) prōdūcit nūlla ambiguitāte, quae iam nōn sit in latīnō. Rēgulae b) c) d) brevī exprimunt fōrmātiōne dē ablātīvō, cum reductiōne dē 4-a dēclīnātiōne ad 2-a, et cum reductiōne ad fōrma ūnica dē 3-a dēclīnātiōne.

---

prōdūcō, -ere, -dūxī, -ductus: (here) bring forth, produce
ambiguitās, -ātis f.: ambiguity
brevī (adv.): briefly, in short
fōrmātiō, -ōnis f.: formation (of a word)
reductiō, -ōnis f.: reduction
dēclīnātiō, -ōnis f.: (here) declination
ūnicus, -a, -um: one

---

nūlla ambiguitāte, quae iam nōn sit in latīnō: *no ambiguity that is not already in Latin*; in other words, any ambiguities that may result from rule a) were already present in Latin; *sit* is subjunctive due to the hypotheticality of the statement (nobody can be certain of all ambiguities present in a language)

4-a dēclīnātiōne: 4-a is an abbreviation of *quārta* (likewise for the other ordinals); Peano replaces the 4th and miscellaneous declensions with the 2nd and 3rd declensions, respectively (e.g. *frūctus, -ūs m.* becomes *frūctō*)

§ 2. — *Genere masculīnō, fēminīnō et neutrō.*

« Discrīmen generis nihil pertinet ad grammaticam ratiōnālem. »

LEIBNIZ.

Nōmen īsolātō nōn habet genere. Quum volumus indicāre ille, scrībēmus explicitē “mās, fēmina.” Ita “māter est *bona*” fit “māter est fēmina bonō”; sed idea dē fēmina iam continētur in māter; igitur post simplificātiōne: “māter est bonō.”

---

genus, generis n.: (here, grammatical) gender
masculīnus, -a, -um: masculine
fēminīnus, -a, -um: feminine
neuter, -tra, -trum: neuter (from *ne-* “not” + *uter* “either of two”)
discrīmen, discrīminis n.: distinction, discrimination, separation
pertineō, -ēre, -uī, – (+ *ad*): pertain to, belong to
grammatica, -ae f.: grammar (from the Greek γραμματική)
ratiōnālis, -e: rational
īsolātus, -a, -um: isolated (borrowed from Italian and in turn derived from the Latin *īnsulātus, -a, -um*, “made into an island”)
quum = cum
explicitē (adv.): explicitly
mās, maris m.: male
fēmina, -ae f.: female
igitur: therefore
simplificātiō, -ōnis f.: simplification

Quum volumus indicāre ille: temporal *cum* clause

“mās, fēmina”: these words usually function as nouns, but they can also serve as as adjectives, with Peano assigning them a fixed gender

Indicātiōne dē genere ēvānēscit saepe in lingua scientifica. In lingua familiāre sufficit cōnservāre genere in ūnō prōnōmen “is, ea, id”, vel in antīquō “hi, hae, ho.” [. . .]

---

indicātiō, -ōnis f.: indication
ēvānēscō, -ere, -vānuī, –: disappear (cf. evanescent)
scientificus, -a, -um: scientific
familiāris, -e: (here) familiar, customary
cōnservō (1): preserve
prōnōmen, prōnōminis n.: pronoun
antīquus, -a, -um: ancient

---

in ūnō prōnōmen “is, ea, id”, vel in antīquō “hi, hae, ho”: Peano’s suggestion is to add the pronoun in front of the noun whose gender one wants to specify; *hice, haece, hoce* were the Old Latin forms of *hic, haec, hoc* that were used as emphatic alternatives in Classical Latin and became particularly frequent in Neo-Latin, oftentimes written incorrectly as *hicce, haecce, hocce* by adding *-ce* to all declensions; Peano therefore reasons that the original versions of *hice, haece, hoce* would lack this *-ce* suffix entirely

Prīncipiō Dē Permanentia: Exercitiō dē Latīnō rēctō

[. . .] Ita Algebra, quī indicā cum idem signō +, ×, = . . . operātiōne nōn sōlō inter numerō integrō, sed etiam inter frāctō, es magnō prōgressō super longō methodō dē antīquō graecō;

---

prīncipium, -iī n.: (here) principle
permanentia, -ae f.: permanence
exercitium, -iī n.: training, exercise
rēctus, -a, -um: (here) correct, proper
algebra, -ae f.: algebra (from the Arabic الْجَبْر *al-jabr* for “setting up,” a term used by the mathematician al-Khwarizmi to refer to a method of solving equations)
signum, -ī n.: sign
operātiō, -ōnis f.: operation
nōn sōlum . . . sed etiam: not only . . . but also
numerus, -ī m.: number
integer, -gra, -grum: (here, in a mathematical context) integral
frāctus, -a, -um: fractional (lit. “broken”)
super (prep. + acc.): over; beyond
prōgressus, -ūs m.: a going forward; advancement, progress
graecus, -a, -um: Greek; (as a substantive plural) Greeks

---

quī indicā cum idem signō +, ×, = . . . sed etiam inter frāctō: relative pronouns like *quī* are no longer declined; verbs are no longer conjugated and instead are all in the singular imperative form; by Peano’s rules, *sōlō* is equivalent to *sōlum*; the plus sign was possibly created by the philosopher and mathematician Nicole Oresme (and, in fact, like the ampersand, came from *et*); the multiplication sign is accredited to William Oughtred, and the equals sign (representing two equal lines) to Robert Recorde; prior to the adoption of these symbols, all algebra required verbose explanations based on geometric reasoning

es: singular imperative form of *esse* and stands for *est*

Algebra redūce tōtō librō X dē Euclīde, nōtō per magnitūdine et difficultāte, ad ūnō pāgina, quī professōre solē docē in ūnō lēctiōne in Īnstitūtō tecnicō (Vidē Fōrmul. p. 111).

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redūcō, -ere, -dūxī, -ductus: (here) reduce

Euclīdēs, is m.: Euclid (Latinization of Εὐκλείδης; see below)

nōtus, -a, -um: known, famous (from *nōscō, -ere, nōvī, nōtus*, “to learn”)

per (prep. + acc.): (here) on account of, for

magnitūdō, -inis f.: greatness, size (cf. magnitude)

difficultās, -ātis f.: difficulty

professor, -ōris m.: professor

lēctiō, -ōnis f.: lecture

īnstitūtum, -ī n.: (here) institute

tecnicus = technicus, -a, -um: technical (from the Greek τεχνικός, -ή, -όν; Peano borrows the spelling from Italian)

Fōrmul. = fōrmulārium, -iī n.: formulary (see below)

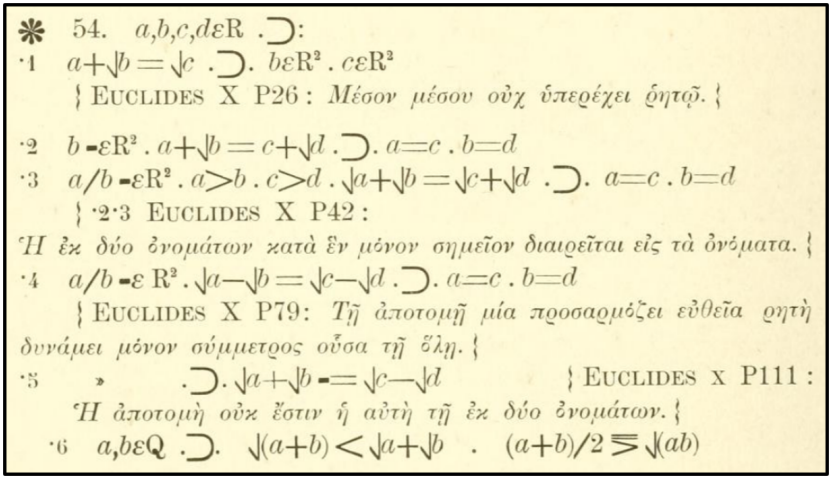
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tōtō librō X dē Euclīde: Book X of Euclid’s *Elements* is devoted to irrational magnitudes, which was a particularly controversial topic in Ancient Greek mathematics because many, including the Pythagoreans, had believed that any two lengths were commensurable to each other (i.e. there always exists a unit such that both lengths are integral multiples of the unit); the mathematical ideas of the book are ascribed to Theaetetus of Athens, whose mentor, Theodorus of Cyrene, showed that the square roots of all non-perfect squares up to 17 are irrational

quī professōre solē docē: *which a professor is accustomed to teach*; the antecedent of *quī* is *ūnō pāgina*; *solē*, a makeshift imperative form of *soleō*, represents a 3rd-person singular present active indicative, while *docē* would be a complementary infinitive in standard Latin, as indicated by the word order alone

Īnstitūtō tecnicō: STEM-based secondary schools in Italy

Fōrmul. p. 111: Peano references his *Fōrmulāriō Mathēmaticō*, a book initially written in French (and then translated into the “Latin without inflections” for its fifth edition) that discusses the fundamentals of higher mathematics in clear and concise notation accessible to anyone with some experience in set theory; an excerpt is shown in Figures 2 & 3



✻ 54. $a,b,c,d \varepsilon \mathrm{R} \;.\supset:$

·1 $a+\sqrt{b}=\sqrt{c} \;.\supset.\; b \varepsilon \mathrm{R}^2 \,.\, c \varepsilon \mathrm{R}^2$

{ Euclides X P26 : *Μέσον μέσου οὐχ ὑπερέχει ῥητῷ.* }

·2 $b \sim \varepsilon \mathrm{R}^2 \,.\, a+\sqrt{b}=c+\sqrt{d} \;.\supset.\; a=c \,.\, b=d$

·3 $a/b \sim \varepsilon \mathrm{R}^2 \,.\, a>b \,.\, c>d \,.\, \sqrt{a}+\sqrt{b}=\sqrt{c}+\sqrt{d} \;.\supset.\; a=c \,.\, b=d$

{ ·2·3 Euclides X P42 :

Ἡ ἐκ δύο ὀνομάτων κατὰ ἓν μόνον σημεῖον διαιρεῖται εἰς τὰ ὀνόματα. }

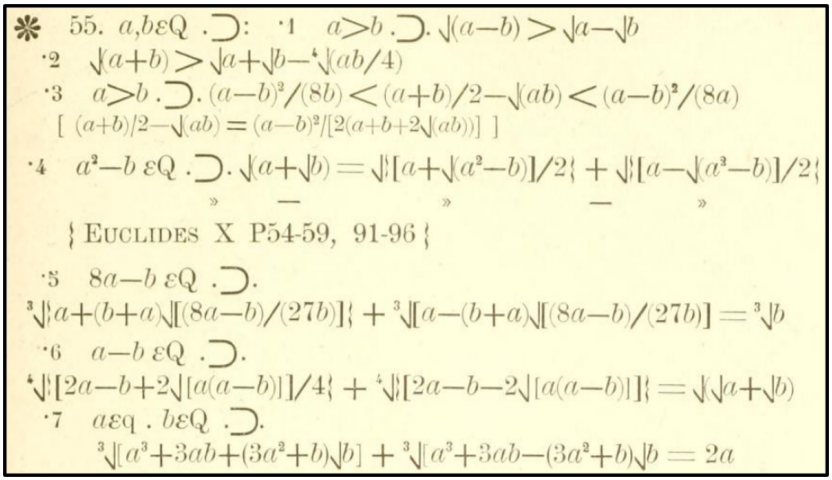
·4 $a/b \sim \varepsilon \mathrm{R}^2 \,.\, \sqrt{a}-\sqrt{b}=\sqrt{c}-\sqrt{d} \;.\supset.\; a=c \,.\, b=d$

{ Euclides X P79: *Τῇ ἀποτομῇ μία προσαρμόζει εὐθεῖα ῥητὴ δυνάμει μόνον σύμμετρος οὖσα τῇ ὅλῃ.* }

·5 » $\;.\supset.\; \sqrt{a}+\sqrt{b} \sim = \sqrt{c}-\sqrt{d}$ { Euclides x P111 :

Ἡ ἀποτομὴ οὐκ ἔστιν ἡ αὐτὴ τῇ ἐκ δύο ὀνομάτων. }

·6 $a,b \varepsilon \mathrm{Q} \;.\supset.\; \sqrt{(a+b)} < \sqrt{a}+\sqrt{b} \quad . \quad (a+b)/2 \geqq \sqrt{(ab)}$



✻ 55. $a,b \varepsilon \mathrm{Q} \;.\supset:$ ·1 $a>b \;.\supset.\; \sqrt{(a-b)} > \sqrt{a}-\sqrt{b}$

·2 $\sqrt{(a+b)} > \sqrt{a}+\sqrt{b}-\sqrt[4]{(ab/4)}$

·3 $a>b \;.\supset.\; (a-b)^2/(8b) < (a+b)/2-\sqrt{(ab)} < (a-b)^2/(8a)$

$[\; (a+b)/2-\sqrt{(ab)} = (a-b)^2/[2(a+b+2\sqrt{(ab)})] \;]$

·4 $a^2-b \;\varepsilon \mathrm{Q} \;.\supset.\; \sqrt{(a+\sqrt{b})} = \sqrt{\{[a+\sqrt{(a^2-b)}]/2\}} + \sqrt{\{[a-\sqrt{(a^3-b)}]/2\}}$

» — » — »

{ Euclides X P54-59, 91-96 }

·5 $8a-b \;\varepsilon \mathrm{Q} \;.\supset.$

$\sqrt[3]{\{a+(b+a)\sqrt{[(8a-b)/(27b)]}\}} + \sqrt[3]{[a-(b+a)\sqrt{[(8a-b)/(27b)]}} = \sqrt[3]{b}$

·6 $a-b \;\varepsilon \mathrm{Q} \;.\supset.$

$\sqrt[4]{\{[2a-b+2\sqrt{[a(a-b)]}]/4\}} + \sqrt[4]{\{[2a-b-2\sqrt{[a(a-b)]}]\}} = \sqrt{(\sqrt{a}+\sqrt{b})}$

·7 $a \varepsilon \mathrm{q} \;.\; b \varepsilon \mathrm{Q} \;.\supset.$

$\sqrt[3]{[a^3+3ab+(3a^2+b)\sqrt{b}]} + \sqrt[3]{[a^3+3ab-(3a^2+b)\sqrt{b}]} = 2a$

Figures 2 & 3: Pages 110 and 111 of Peano's *Fōrmulāriō Mathēmaticō* restate algebraically a couple of the propositions in Euclid's Book X. $\supset$ is equivalent to the modern implication symbol $\Rightarrow$. Peano explains that $p \supset q$, for statements p and q, is just a reversal of $q \mathrm{C} p$, where C signifies *cōnsequentia*. Image credits: *Formulaire de Mathématiques*, public domain.

In saeculō ultimō plūre auctōre indicā cum idem signō +, sīve summa dē algebra, sīve resultante dē vectōre, et ita cōnstrue Calculō geōmetricō, quī in respectō ad Geōmetria, es idem prōgressō quam Algebra in respectō dē Arithmētica graecō. Tunc prīncipiō dē permanentia idem es ac prīncipiō generāle dē oeconomia, quī subsiste in linguistica, didactica et polītica, ut dēmōnstrā E. Mach, in capitulō dē «nātūra oeconomicō dē prōgressō physicō» dē *Populär Vorlesungen*, Leipzig a. 1903.

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ultimus, -a, -um: last

plūs, plūris: more; very many (comparative of *multus, -a, -um*)

auctor, -ōris m.: author, writer

sīve . . . sīve: either . . . or

summa, -ae f.: sum

resultāns, -antis m.: (in math, as a sum of vectors) resultant

vector, -ōris m.: (in math) vector

cōnstruō, -ere, -strūxī, -strūctus: construct

calculus, -ī m.: calculation, calculus (originally the diminutive of *calx, calcis f.*, "stone," because little stones were used for counting boards)

geōmetricus, -a, -um: geometrical

in respectū (+ gen. or *ad*): with respect to

geōmetria, -ae f.: geometry

quam: (here, in a comparison) as

arithmētica, -ae f.: arithmetic (from the Greek ἀριθμός for "number")

ac: (here, in a comparison) as

generālis, -e: general

oeconomia, -ae f.: economy (from the Greek roots οἶκος, "house," and νόμος, "law"; here used in the more modern usage of the term, as "a careful use of resources" or "thrift")

subsistō, -ere, -stitī, –: (here) abide, remain, exist

linguistica, -ae f.: linguistics

didactica, -ae f.: didactics, the science of teaching (from the Greek διδακτικός, -ή, -όν, ultimately from διδάσκειν, "to teach")

polītica, -ae f.: politics (from the Greek πολιτικός, -ή, -όν, ultimately from πόλις, "city")

dēmōnstrō (1): demonstrate, prove

E. Mach: Ernst Mach (see below)

capitulum, -ī n.: chapter (diminutive of *caput, capitis n.*)

oeconomicus, -a, -um: economical (here, again, in the sense of "thrifty")

physicus, -a, -um: of physics (from the Greek φύσις, "nature")

***Populär Vorlesungen*:** under the title *Populär-wissenschaftliche Vorlesungen*, Ernst Mach published a collection of lectures on various topics in science, including the hair cells of the ear, the speed of light, the purpose of binocular vision, the importance of randomness and accidents in the development of inventions, and the principle of economical thought, which Peano cites here (Mach, 1903)

Glossary

ā/ab (prep. + abl.): by; from; away from
ac/atque: and, also
accipiō, -ere, -cēpī, -ceptus: take, receive, accept
ad (prep. + acc.): to, toward, at
agō, -ere, ēgī, āctus: drive, lead; do; act
aliquis, aliquid: anyone/anything; someone/something
alius, alia, aliud: other, another
alter, -era, -erum: either one (of two), the other, another
altus, -a, -um: high; deep
ambulō (1): walk
amīcus, -ī m.: friend, loved one
animus, animī m.: mind; intellect; soul
annus, -ī m.: year
aqua, -ae f.: water
at: but
audiō, -īre, -īvī, -ītus: hear, listen
aut: or; either . . . or (*aut . . . aut*)
autem: but, however; moreover
benē (adv.): well
bonus, -a, -um: good
brevis, -e: short
caelum, -ī n.: sky, heavens
capiō, -ere, cēpī, captus: seize
caput, capitis n.: head
causa, -ae f.: cause, reason
cīvis, -is m./f.: citizen
corpus, -oris n.: body
cum: with (prep. + abl.); when, while, since, although (conjunction)
dē (prep. + abl.): from; down from; about, concerning, regarding
dēbeō, -ēre, -uī, -itus: owe; must
deinde/dein: then
dēserō, -ere, -uī, -tus: leave, desert, abandon
deus, -ī m.: god
dīcō, -ere, dīxī, dictus: say; call
diēs, diēī m.: day

difficilis, -e: difficult
dō, dare, dedī, datus: give
doceō, -ēre, -uī, -tus: teach
domus, -ī/-ūs f.: house
dūcō, -ere, dūxī, ductus: lead
dum: while
duo, duae, duo: two
dūrus, -a, -um: harsh
ē/ex (prep. + abl.): out of; from
ego, meī, mihi, mē, mē: I, my, me
enim: for, indeed
eō, īre, iī, itus: go
ergō: therefore
et: and; also; even; both . . . and (*et . . . et*)
etiam: also, even
exeō, -īre, -iī, -itus: go forth
facilis, -e: easy
faciō, -ere, fēcī, factus: do, make
ferō, ferre, tulī, lātus: carry; take; endure
fīō, fierī, factus sum: be made, be done; become; happen
genus, generis n.: kind, class, family
gravis, -e: heavy, severe
habeō, -ēre, -uī, -itus: have, hold, consider
hic, haec, hoc: this; these
hīc: here
homō, hominis m.: man, person, human
iam: now; already; at last
īdem, eadem, idem: the same (person or thing)
ille, illa, illud: that; those
in: in, on (prep. + abl.); into, onto (prep. + acc.)
īnsula, -ae f.: island
inter (prep. + acc.): between, among
ipse, -a, -um: him-, her-, itself; the very
is, ea, id: he/she/it/they
iste, ista, istud: this, that
ita: thus, so, like
itaque: and so, therefore

labor, -ōris m.: work, toil, labor
legō, -ere, lēgī, lēctus: read
liber, librī m.: book
lingua, -ae f.: language
locus, -ī m.: place, point
longus, -a, -um: long, far
magis (adv.): more
magnus, -a, -um: great
maior, maius: greater
manus, -ūs f.: hand
mare, -is n.: sea
māter, mātris f.: mother
maximus, -a, -um: greatest, largest
mēns, mentis f.: mind; reason, judgement
meus, -a, -um: my
mīlle, (pl.) mīllia (mīlia): thousand
minor, minus: less, smaller
miser, -era, -erum: wretched, miserable
modus, -ī m.: manner, way, method; measure; limit
mōns, montis m.: mountain
mors, mortis f.: death
multus, -a, -um: much, many
nam: for, because
nātūra, -ae f.: nature
nē: not; that not (negation with subjunctives)
neque/nec: nor, and not; neither . . . nor (*neque . . . neque*)
nihil (indecl.): nothing
nisi: if not, unless; except
nōn: not
nōs, nostrī/nostrum, nōbīs, nōs, nōbīs: we, our, us
noster, -stra, -strum: our
novus, -a, -um: new
nox, noctis f.: night
nūllus, -a, -um: no, none, not any
nunc: now
oculus, -ī m.: eye
omnis, -e: all, every, whole

ostendō, -ere, -ī, ostentus: to show, present
pars, partis f.: part, portion; region
parvus, -a, -um: small
paucus, -a, -um: few
per (prep. + acc.): through; by means of
petō, -ere, -īvī, -ītus: seek
pōnō, -ere, posuī, positus: put, place
possum, posse, potuī: be able, can
post: later (adv.); after (prep. + acc.)
prīmus, -a, -um: first
prō (prep. + abl.): for, in place of; in proportion to, in accordance with
propter (prep. + acc.): because
pulcher, -chra, -chrum: beautiful
putō (1): think
quaerō, -ere, -sīvī, -sītus: seek; desire; require
quam: than (after comparatives)
quārtus, -a, -um: fourth
quattuor (indecl.): four
-que: and (enclitic)
quī, quae, quod: who, which, that
quia: since, because
quīdam, quaedam, quoddam: a certain person/thing
quidem: indeed, certainly
quīnque (indecl.): five
quis, quid: who, what
quisque, quaeque, quodque: each, every, any
quisque, quidque: everyone, everything
quisquis, quidquid: whoever; whatever
quod: because
quoque: too, also
rēs, reī f.: thing; matter, affair
respondeō, -ēre, -ī, -ōnsus: respond, answer
saepe: often
scrībō, -ere, scrīpsī, scrīptus: write
secundus, -a, -um: second
sed: but

semper: always, ever
sequor, -ī, secūtus sum: follow
sī: if
sīc: in this way, thus
sine (prep. + abl.): without
sōl, sōlis m.: sun
soleō, -ēre, -itus sum: be accustomed
sōlus, -a, -um: only, alone
sub (prep. + acc. or abl.): under
sui, sibi, sē, sē: him/her/itself, themselves
sum, esse, fuī, futūrus: be; exist
summus, -a, -um: highest, greatest
suus, -a, -um: his own, her own, its own
tam: so, so much
tamen: nevertheless
tantus, -a, -um: such great, so great
tempus, -oris n.: time
teneō, -ēre, -uī, -tus: hold, keep
terra, -ae f.: land; (in the singular or plural) the Earth, world
tertius, -a, -um: third
tōtus, -a, -um: whole, entire
trēs, tria: three
tū, tuī, tibi, tē, tē: you, your (singular)
tum: then, next; at that time; also
tunc: then, at that time
ubi: where; when
ūnus, -a, -um: one
urbs, urbis f.: city
ut: to, in order to (+ subjunctive); how, when, as (+ indic.)
vel: or; either . . . or (*vel . . . vel*)
veniō, -īre, vēnī, ventus: come
verbum, -ī n.: word
via, -ae f.: road, path
videō, -ēre, vīdī, vīsus: see; seem (pass.)
vir, virī m.: man
vīta, -ae f.: life
vīvō, -ere, vīxī, vīctus: live

vocō (1): call
volō, velle, voluī, –: wish, want; be willing
vōs, vestrī/vestrum, vōbīs, vōs, vōbīs: you, your (plural)